Math 225: Practice Exam 1

This exam is meant to cover the types of problems that you might encounter on Friday's Midterm. It is by no means exhaustive. To get the most out of your studies, you should also review our past quizzes, homeworks, and class notes. Come prepared with questions on this for Wednesday.

1. Give parametric equations for the line that contains the points $(1, 3)$ and $(2, -1)$.
2. Find the points at which the curve $r(t) = \langle t^2, t^3 - 3t \rangle$ has a horizontal tangent.
3. Let $A = (2, 3, 1)$, $B = (6, 1, 5)$, and let $P = (x, y, z)$
   (a) Find the distance from A to B.
   (b) Write the following statement as an equation:

   The distance from P to A is the same as the distance from P to B.

   (c) Simplify your equation and describe the set of points (x, y, z) that satisfy the description in part (b).
4. Consider the following two line equations:

$$\ell_1(t) = \langle 1 + 3t, t, 2 - 2t \rangle;\ \ell_2(t) = \langle -6t + 3, -2t + 3, 4t \rangle$$

   (a) Explain why ℓ_1 is parallel to ℓ_2.
   (b) Find the plane that contains ℓ_1 and ℓ_2.
5. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors. Prove that if $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$, then $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.
6. Suppose that we have three lines, each of which is skew to the other two. Might there be a plane parallel to all 3 lines? Must there be a plane parallel to all 3 lines? Explain.
7. Let $x = t^2 + 3t + 1$ and $y = 2t - t^2$.
   (a) Find the values of t for which the above curve has horizontal and vertical tangents.
   (b) Find the area bound by this curve and the $x-$axis. (You'll need to find the t values where the curve meets this axis).
8. Consider the equation $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos(2t) \rangle$.
   (a) This curve is the intersection of which two surfaces? (Here, you'll need algebraic relations between x, y and z components. Try a trig identity for the relation with z, which I can 'sell' to you if necessary.) Be sure that you give the names of the surfaces as well as the algebraic equations.
   (b) Give a rough sketch of the curve. (You may give a prose description to aid you here.)
   (c) Find the equation of the tangent line to $\mathbf{r}(t)$ when $t = \frac{\pi}{2}$.

9. Suppose that the position of an object is given by $\mathbf{r}(t) = \langle t^2, e^t, te^t \rangle$.

   (a) Find the velocity and acceleration of the particle as a function of time.

   (b) Find the speed of the particle as a function of time. Is the speed ever 0?

10. Match the equation to the surface description. Warning: There is one outlier in each group!!

(a) $x^2 + 9y^2 + 81z^2 = 81$	I Hyperboloid of One Sheet
(b) $x^2 + 9y^2 + 81 = 81z^2$	II Hyperboloid of Two Sheets
(c) $x^2 + 9y^2 + 81 = 81z$	III Ellipsoid
(d) $x^2 - 9y^2 + 81 = 81$	IV Elliptical Paraboloid

Remember also to review properties of vector projections and vector components.