A B

1) (1,3) and (2,-1)

$x = x_0 + \Delta x\, t = 1 + t$

$y = y_0 + \Delta y\, t = 3 + (-4t)$

$\overrightarrow{AB} = \langle 1, -4 \rangle$

$l(t) = \langle 1,3 \rangle + t\langle 1,-4 \rangle$

3) $A = (2,3,1)$

$B = (6,1,5)$

$P = (x,y,z)$

$d(A,B) = \sqrt{(6-2)^2 + (1-3)^2 + (5-1)^2}$

$= \sqrt{4^2 + 2^2 + 4^2} = \sqrt{36} = 6$

$d(P,A) = d(P,B) \quad \sqrt{(x-2)^2 + (y-3)^2 + (z-1)^2} = \sqrt{(x-6)^2 + (y-1)^2 + (z-5)^2}$

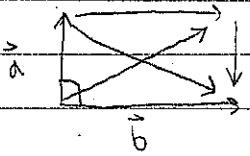
$(x-2)^2 + (y-3)^2 + (z-1)^2 = (x-6)^2 + (y-1)^2 + (z-5)^2$

$x^2 - 4x + 4 + y^2 - 6y + 9 + z^2 - 2y + 1 = x^2 - 12x + 36 + y^2 - 2y + 1 + z^2 - 10z + 25$

$8x - 4y + 8z = 48$

$2x - y + 2z = 12$

5) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then $\vec{a} \perp \vec{b}$



$|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$

$(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b})$

$|\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 = |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$

$4\vec{a} \cdot \vec{b} = 0$

$\vec{a} \cdot \vec{b} = 0$

$* \; |\vec{v}|^2 = \vec{v} \cdot \vec{v} \; *$

7) $x = t^2 + 3t + 1$

$y = 2t - t^2$

a. $\frac{dy}{dx} = \frac{2-2t}{2t+3}$ vertical when $t = 3/2$

horizontal when $t = 1$

b. $y = 0 = 2t - t^2$ @ $t = 0$ and $t = 2$

y dx/dt

$$\int_0^2 (2t - t^2)(2t + 3)\,dt$$

$$\int_0^2 (6t - 2t^3 + t^2)\,dt$$

$$3t^2 - \frac{t^4}{2} + \frac{t^3}{3}\Big|_0^2$$

$12 - 8 + \frac{8}{3} = \boxed{20/3}$

9) $\vec{r}(t) = \langle t^2, e^t, te^t \rangle$

$\vec{r}'(t) = \langle 2t, e^t, e^t + te^t \rangle$

$\vec{r}''(t) = \langle 2, e^t, te^t + 2e^t \rangle$

b. Speed $= |\vec{r}'(t)| = \sqrt{4t^2 + (e^t)^2 + (te^t + e^t)^2}$ ← never zero

smooth curve

even numbers:

2. $\vec{r}(t) = \langle t^2, t^3 - 3t \rangle$ has a horizontal tangent

$$\text{slope} = \frac{dy}{dx} \rightarrow \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2-3}{2t}$$

horizontal → slope = 0 $\longrightarrow 3t^2 - 3 = 0$

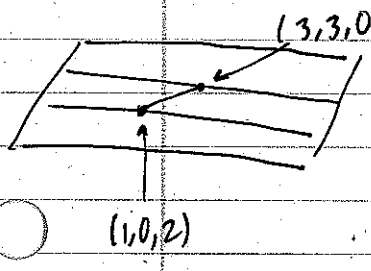
$t = \pm 1$

points $(1,-2)$ & $(+1,2)$

* vertice tangent @ $t=0$ *

4. $l_1 = \langle 1+3t, t, 2-2t \rangle$ $\quad \vec{V}_1 = \langle 3, 1, -2 \rangle$

$l_2 = \langle -6t+3, -2t+3, 4t \rangle$ $\quad \vec{V}_2 = \langle -6, -2, 4 \rangle$ → multiple by a scalar



$$\langle 3, 1, -2 \rangle \times \langle 2, 3, -2 \rangle = \langle 4, 2, 7 \rangle$$

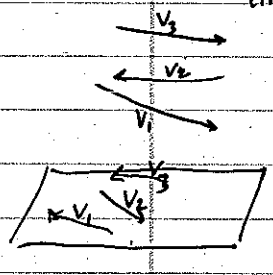
$$\text{plane} = 4(x-3) + 2(y-3) + 7z = 0$$

two lines in a plane → cross product of direction vector to get normal vector

* understand equations for a line verse a plane *

6. you can have 3 lines parallel to a plane

→ circumstance: lines have to be flat to the plane but skew to the other lines



$\cos 2t = \cos^2 t - \sin^2 t$
$2\cos^2 t - 1$
$1 - 2\sin^2 t$

8. $\vec{r}(t) = \langle \cos t, \sin t, \cos 2t \rangle$

$x^2 + y^2 = 1$ cylinder

hyp. paraboloid $x^2 - y^2 = z$

parabolic cylinder $2x^2 - 1 = z$

parabolic cylinder $1 - 2y^2 = z$

cylinder + hyp paraboloid → pringle shape

∞ "border"

tangent line when $t = \pi/2$

$\vec{r}(\pi/2) = \langle \cos(\pi/2), \sin(\pi/2), \cos \pi \rangle$

$\vec{r}'(t) = \langle -\sin t, \cos t, -2\sin 2t \rangle$

$\vec{r}'(\pi/2) = \langle -\sin \pi/2, \cos(\pi/2), -2\sin \pi \rangle$

$\langle -1, 0, 0 \rangle$

line: $\vec{l}(t) = \langle 0, 1, -1 \rangle + t\langle -1, 0, 0 \rangle$

10. a.) $x^2 + 9y^2 + 81z^2 = 81$ ellipsoid

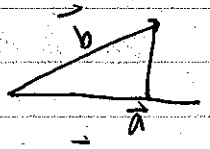
b.) $x^2 + 9y^2 + 81 = 81z^2$ hyperboloid of 2 sheets

$z \geq 1$

c.) $x^2 + 9y^2 + 81 = 81z$ elip. paraboloid

d.) $x^2 - 9y^2 + 81 = 81$ degenerate parabola

$x = \pm 3y$



$\text{proj}_{\vec{a}} \vec{b}$

$\text{proj}_{\vec{a}} \vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$

$\text{comp}_{\vec{a}} \vec{b} = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}|}$

what to hand in on Friday:

12.6 # 23-26

13.1 # 15, 29, 43

13.2 # 12, 25