Math 225: Exam the Second

ElevenElevenEleven

This exam is closed book and closed notes. READ ALL DIRECTIONS CAREFULLY!! Please justify all of your answers. You may use a calculator for arithmetic and scienific functions only (ie, no graphing). You have two hours.

1. Let $f(x,y) = \sqrt{9-x^2-y^2}$
   (a) What is the domain of f?
   (b) For which k can we draw level curves of the form $f(x,y) = k$?
   (c) Draw level curves below for $k = 0, 1, 2, 3$, with attention to spacing.
2. Find the mixed partial derivative f_{xy} if $f(x,y) = x^2 \arccos(x) + xy + y^y$.
3. Let $f(x,y) = e^{xy}$.
   (a) Find the tangent plane at the point (2,1), and use it to approximate $f(2.1, 1.04)$. (Please express your answer as a multiple of e^2.
   (b) Find $D_u(f)$ as we move from $(2,1)$ to $(5,5)$.
4. Let $x^2yz + yz + x^3y = 5$.
   (a) Find $\frac{\partial y}{\partial x}$ and $\frac{\partial y}{\partial z}$.
   (b) Find the tangent plane to $f(x,y)$ at (-1,1,3)
5. Let $f(x,y) = x^2 + kxy + y^2$, where k is a constant.
   (a) Find the critical point of f (it should be independent of k).
   (b) For which values of k is the critical point a minimum?
   (c) For which values of k is the critical point a maximum?
   (d) For which values of k is the critical point a saddle point?
   (e) For which values of k does the discriminant test fail to be conclusive at the critical point?
   (f) What is the nature of the critical point for those values of k?
6. Find the maximum and minimum values of $f(x,y) = 2x+y$ subject to the constraint $x^2+2y^2 = 1$. You may use any method that you wish.
7. Find
$$\iint_R x(xy+1)^3 \, dA$$
where $R = [0,1] \times [0,2]$.
8. Consider the region R bound by the lines $y = 0$, $x = 9$ and $x = y^2$. Write
$$\iint_R f(x,y) \, dA$$
in both possible orders. (That is, use R to determine the bounds of integration).

9. Let $z^2 = 1 + x^2 + y^2$.

   (a) What quadric surface is the graph of this equation?

   (b) Find the volume in the first octant bound by this surface and the plane $z = 3$.

10. Convert

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x + y \, dy \, dx$$

    to polar coordinates, and solve the integral.

11. Set up the integral to find the area of the surface of $f(x, y) = x^2 + 2y^2$ over the triangle with vertices $(0, 0), (1, 1)$ and $(1, -1)$. You need not compute this integral.

12. Should you get the same integral for the area of the surface of $f(x, y) = x^2 + 2y^2 + 3$? Explain both geometrically and algebraically.