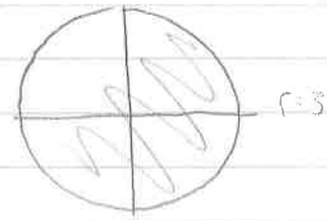
1) $f(x,y) = \sqrt{9-x^2-y^2}$



$9-x^2-y^2 \geq 0 \qquad 9 \geq x^2+y^2$

$k = \sqrt{9-x^2-y^2}$

$3 \geq k \geq 0$

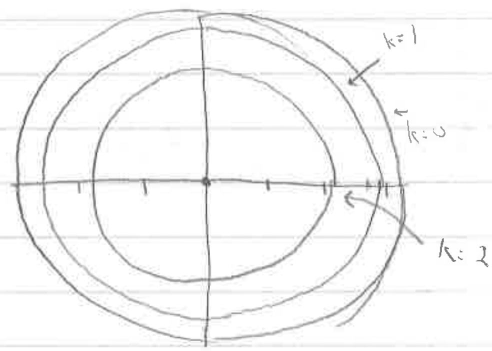
Draw level curves k: 0, 1, 2, 3 with attention to spacing

$0 = \sqrt{9-x^2-y^2} \qquad 9 = x^2+y^2$

$1 = \sqrt{9-x^2-y^2} \qquad 8 = x^2+y^2$

$2 = \sqrt{9-x^2-y^2} \qquad 5 = x^2+y^2$

$3 = \sqrt{9-x^2-y^2} \qquad 0 = x^2+y^2$



$x^2+y^2+z^2=9 \qquad z>0$

Sphere of radius 3

2) $f(x,y) = x\arccos(x) + xy + y^y$

only mixed terms contribute to mixed derivatives

$f_x =$ all in terms of x $+\ y + 0$

$f_{xy} = 1$

3) $f(x,y) = e^{xy}$ <u>Tangent Planes</u>

at (2,1)

Explicit: $z = f(x,y)$ ← functions

T-plane: $z - z_0 = f_x(x - x_0) + f_y(y - y_0)$

Implicit: Not necessarily functions

$F(x,y,z) = K$

$F_x(x - x_0) + F_y(y - y_0) + F_z(z - z_0) = 0$

$\langle F_x, F_y, F_z \rangle = \vec{\nabla} F$

$f_x = ye^{xy}$ @ $(2,1) = e^2$ $\qquad f(2,1) = e^2 = z_0$

$f_y = xe^{xy}$ @ $(2,1) = 2e^2$

$$\text{T-Plane}: z - \overset{f(x,y)}{e^2} = e^2(x-2) + 2e^2(y-1)$$

approximate $f(2.1, 1.04)$

$z = e^2 + e^2(2.1 - 2) + 2e^2(1.04 - 1)$ $\qquad \vec{\nabla} f = \langle e^2, 2e^2 \rangle$

$z = e^2(1 + 0.1 + 0.08)$

$z = 1.18e^2$

from (2,1) towards (5,5) $\quad \vec{v} = \langle 3, 4 \rangle \quad \vec{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

$\vec{u} \cdot \vec{\nabla} f = \frac{3}{5}e^2 + \frac{8}{5}e^2 = \frac{11}{5}e^2$ ← must be a scalar

increasing because positive

$F(x,y,z) = k$

4) $x^2yz \underset{+yz}{\downarrow} + x^3y = 5$

$\dfrac{dy}{dx} = \dfrac{-F_x}{F_y}$ $\qquad$ $\dfrac{dy}{dz} = \dfrac{-F_z}{F_y}$

$\dfrac{dy}{dx} = \dfrac{-(2xyz + 3x^2y)}{x^2z + z + x^3}$ $\qquad$ $\dfrac{dy}{dz} = \dfrac{-(x^2y + y)}{x^2z + z + x^3}$

Find tangent plane

$F_x = 2xyz + 3x^2y = -6 + 3 = -3$

$F_y = x^2z + z + x^3 = 3 + 3 + -1 = 5$

$F_z = x^2y + y = 2$

T-plane $\qquad -3(x+1) + 5(y-1) + 2(z-3) = 0$

5) $f(x,y) = x^2 + kxy + y^2$

$f_x = 2x + ky = 0$

@ $(0,0)$ = c.p.

$f_y = kx + 2y = 0$

$f_{xx} f_{yy} - (f_{xy})^2$ $\quad >0$ max or min

$(2)(2) - (k)^2 = 4 - k^2$ $\quad <0$ saddle point

$=0$ not determinible

$4 - k^2 \quad 4 > k^2 \Rightarrow -2 < k < 2 \quad |k| < 2 = \text{min}$

never have a maximum

saddle point when $|k| > 2$

inconclusive when $|k| = 2$

$k=2 \quad f(x,y) = x^2 + 2xy + y^2 = (x+y)^2$ parabolic cylinder

$k=-2 \quad f(x,y) = x^2 - 2yx + y^2 = (x-y)^2$

6) max & min of $f(x,y) = 2x + y$ s.t. $x^2 + 2y^2 = 1$

$f_x = 2 = \lambda 2x \qquad x = \frac{1}{\lambda}$ $\qquad\qquad \vec{\nabla} f = \lambda \vec{\nabla} g$

$f_y = 1 = \lambda 4y \qquad y = \frac{1}{4\lambda}$

$x^2 + 2y^2 = 1$

$\frac{1}{\lambda^2} + \frac{2}{16\lambda^2} = 1 \qquad \frac{9}{8\lambda^2} = 1 \qquad \lambda^2 = \frac{9}{8} \qquad \lambda = \pm\sqrt{9/8} = \pm\frac{3}{2\sqrt{2}} = \lambda$

positive $\quad x = \frac{2\sqrt{3}}{3} \qquad y = \frac{\sqrt{2}}{6}$

negative $\quad x = \frac{-2\sqrt{3}}{3} \qquad y = \frac{-\sqrt{2}}{6}$

maximum $= \frac{4\sqrt{2}}{3} + \frac{\sqrt{2}}{6} = \frac{9\sqrt{2}}{6} = \frac{3\sqrt{2}}{2}$

7) $\iint_R x(xy+1)^3\, dA \qquad [0,1] \times [0,2]$

$\int_0^1 \int_0^{2x+1} x(xy+1)^3\, dy\,dx$

$u = xy+1 \qquad 2x+1$

$du = x\,dy \qquad 0x+1 = 1$

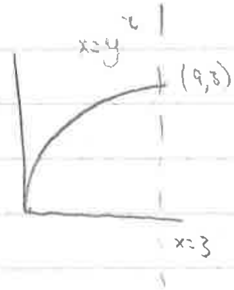
$\int_0^1 \int_0^{2x+1} u^3\, du\,dx$

$\int_0^1 \frac{u^4}{4} \Big|_1^{2x+1} dx$

$\int_0^1 \frac{(2x+1)^4}{4} - \frac{1}{4}\, dx$

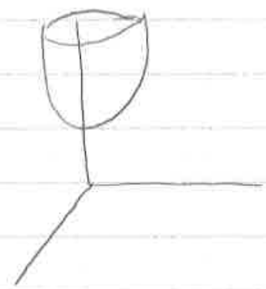
$\frac{(2x+1)^5}{40} - \frac{1}{4}x \Big|_0^1 = \frac{3^5}{40} - \frac{1}{4} - \frac{1}{40}$

8) $\int_0^3\int_{y^2}^9 dx\,dy$



$\int_0^9\int_0^{\sqrt{x}} dy\,dx$

9) $z^2 = 1+x^2+y^2$ hyperboloid of 2 sheets



volume in first octant and $z=3$

$\int_0^{\pi/2}\int_0^{2\sqrt{2}} 3-\sqrt{1+x^2+y^2}$

$9 = 1+x^2+y^2$

$r=(\sqrt{2})2$

$\int_0^{\pi/2}\int_0^{2\sqrt{2}}\left(3-\sqrt{1+r^2}\right) r\,dr\,d\phi$

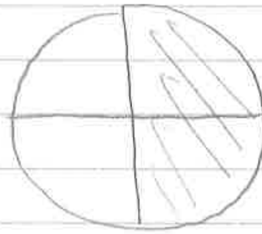
$\int_0^{\pi/2}\int_0^{2\sqrt{2}} 3r - r\sqrt{1+r^2}\,dr\,d\phi$

$\int_0^{\pi/2}\left[\frac{3}{2}r^2 - \frac{1}{3}\left(1+r^2\right)^{3/2}\right|_0^{2\sqrt{2}} d\phi$

$\int_0^{\pi/2} 12-\frac{1}{3}\sqrt{27}+\frac{1}{3}$

$\int_0^{\pi/2}\frac{36-27+1}{3}\,d\phi = \frac{\pi}{2}\left(\frac{36-27+1}{3}\right)$

$$\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x+y \, dy\, dx$$



$$\int_{-\pi/2}^{\pi/2} \int_0^2 (r\cos\phi + r\sin\phi)\, r\, dr\, d\phi$$

$$\int_{-\pi/2}^{\pi/2} \int_0^2 r^2\cos\phi + r^2\sin\phi \, dr\, d\phi$$

$$\int_{-\pi/2}^{\pi/2} \int_0^2 \frac{r^3}{3}\cos\phi + \frac{r^3}{3}\sin\phi \Big|_0^2 \, d\phi$$

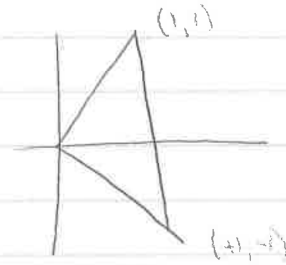
$$\int_{-\pi/2}^{\pi/2} \frac{8}{3}(\cos\phi + \sin\phi)\, d\phi$$

$$= \frac{8}{3}(\sin\phi - \cos\phi)\Big|_{-\pi/2}^{\pi/2}$$

$$= \frac{8}{3}(1-0-(-1-0)) = \frac{16}{3}$$

10) $f(x,y) = x^2 + 2y^2$

$\iint \sqrt{1+(f_x)^2+(f_y)^2}$



$\iint \sqrt{1+4x^2+16y^2}\, dA$

$\int_0^1 \int_{-x}^{x} \sqrt{1+4x^2+16y^2}\, dy\, dx$

the second part is exactly the same