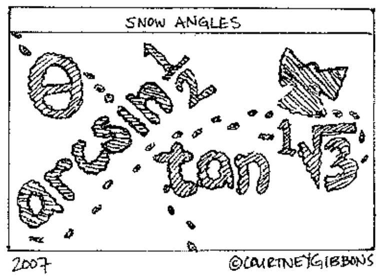
Math 225: Quiz the First
January 31, 2014

KEY

This quiz is closed book and closed notes. You may not use a calculator on this quiz. Please justify all of your answers. You have until the end of the class period to finish.



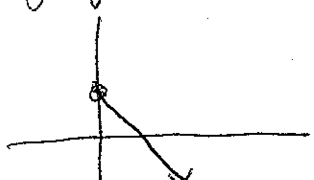
1. Suppose $(x(t), y(t))$ are a set of parametric equations for $a \leq t \leq b$. To plot the same curve in the reverse direction, what algebraic operation do we need to perform?

Replace (t) with $(-t)$

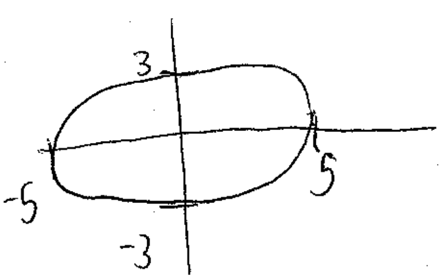
That is, simplify $x(-t)$
$y(-t)$
$a \leq -t \leq b$.

2. What (if any) are the differences between the plots of the line $y = 1 - x$ and the parametric curve $(x = e^t, y = 1 - e^t)$?

$x = e^t$
$y = 1 - e^t$ is a line along $y = 1 - x$, but only takes on positive x values!



3. Give the parametric equations for an ellipse centered at the origin with horizontal axis length 10 and vertical axis length 6.



$x = 5\cos t$
$y = 3\sin t$
$(0 \leq t \leq 2\pi)$

4. Consider the parametric equations $x = 1 - 2t$, $y = 3t - t^2$.

(a) Find the *point* at which the curve has a horizontal tangent.

Tangent line

$$\text{Slope} = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3-2t}{-2}$$

horiz tan $\Rightarrow$ slope $= 0$ $\Rightarrow$ $t = \frac{3}{2}$

$$\text{point} = \left(1 - 2\left(\tfrac{3}{2}\right),\ 3\left(\tfrac{3}{2}\right) - \left(\tfrac{3}{2}\right)^2\right)$$

$$= \left(-2,\ 9/4\right)$$

(b) Does the curve ever have a vertical tangent? Explain.

No. Since $\frac{dx}{dt} = -2 \neq 0$, we never have an "undefined" slope.

(c) Does the curve move left-to-right, or right-to-left? Explain.

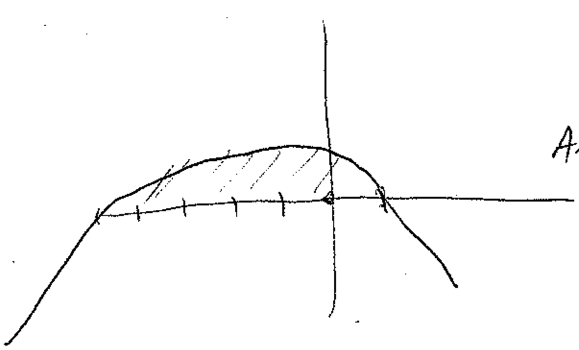
Since x decreases as t increases, the curve moves

right to left

(d) Find the area bound by the curve and the x-axis.

$y = 0 \Rightarrow 3t - t^2 = 0 \Rightarrow t = 0, 3$

opp movement $\qquad x = 1$ or $x = -5$



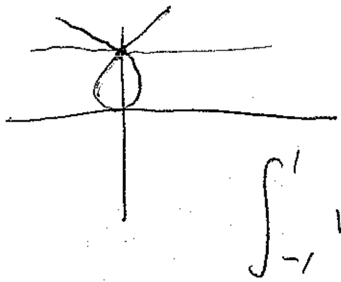
$$\text{Area} = \int_a^b y\,dx = \int_3^0 (3t - t^2)(-2)\,dt$$

$$= 2\int_0^3 3t - t^2\,dt = 2\left[\frac{3t^2}{2} - \frac{t^3}{3}\right]_0^3$$

$$= 2\left[\frac{27}{2} - \frac{27}{3}\right] = 2\left[\frac{27}{6}\right]$$

$$= \boxed{9}$$

5. In class, we examined the 'teardrop' given by $x = t^3 - t, y = t^2$. SET UP, but DO NOT EVALUATE the integral that gives the length of the 'teardrop' portion of the curve (that is, the length below the line $y = 1$). You do not need to simplify within the radical.



$t = -1 \leftarrow x = 0, \; y = 1$

$t = 1$

$$\int_{-1}^{1} \sqrt{(x'(t))^2 + (y'(t))^2}\, dt = \int_{-1}^{1} \sqrt{(3t^2-1)^2 + (2t)^2}\, dt.$$

6. Consider the sphere $x^2 + y^2 + z^2 - 4x + 2y - 2z + 3 = 0$.

(a) Find the center and the radius of this sphere.

$$x^2 - 4x + y^2 + 2y + z^2 - 2z = -3$$

$$x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 - 2z + 1 = -3 + 4 + 1 + 1$$

$$(x-2)^2 + (y+1)^2 + (z-1)^2 = 3$$

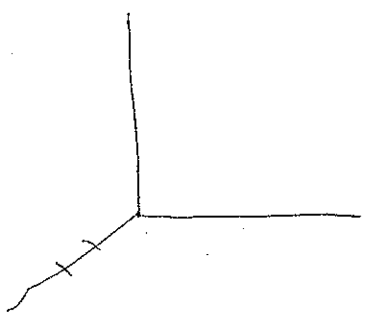
Center $(2, -1, 1)$ radius $= \sqrt{3}$.

(b) Does this sphere intersect the xy-plane? What about the xz-plane or the yz-plane? Explain.

$z = 0 \downarrow$ yes

$y = 0 \downarrow$ yes

$x = 0 \downarrow$ no.



if $x = 0$

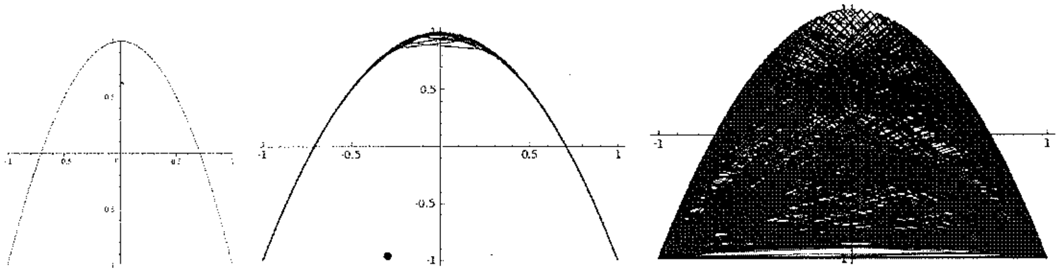
then

$4 + (y+1)^2 + (z-1)^2 = 3$

or $(y+1)^2 + (z-1)^2 = -1$

which is not permissable.

7. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t)$, which we showed in class to be a parabola similar to $y = 1 - 2x^2$, but restricted and periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 2\pi)$, $(0, 100\pi)$, and $(0, 500\pi)$ The 'dot' in the second graph is merely a printing error).

The "impurities" arise because the computer is connecting dots to form line segments to graph, and these get more "spread out" in a longer domain.