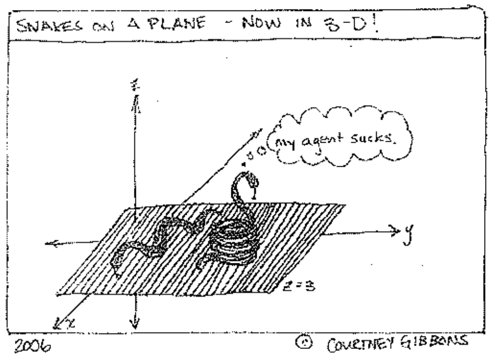
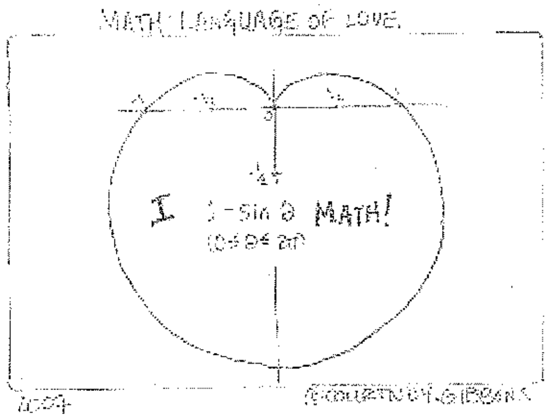
KEY

Math 225: Quiz the Third

February 14, 2014

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.





1. Identify the following surfaces, *and* answer the related questions.

   (a) $2x^2 + 32y^2 + 72z^2 = 288$ (Give maximum values for x, y, and z).

   Ellipsoid

   $$\frac{x^2}{144} + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

   max $x = 12$
   max $y = 3$
   max $z = 2$

   (b) $2x^2 - 72y^2 + 32z^2 = 288$ (Give permissible values for y)

   Hyp of one sheet

   $2x^2 + 32z^2 = 288 + 72y^2$ All real y allowed

   (c) $2x^2 - 32y^2 - 72z^2 = -288$ (Give permissible values for x)

   Hyp of one sheet

   $2x^2 + 288 = 32y^2 + 72z^2$ All real x allowed

   (oops!)

   (d) $2x + 18y^2 + 32z^2 = 0$ (Give the axis of symmetry and 'direction' of this graph)

   $2x = -18y^2 - 32z^2$ Paraboloid, down the x-axis

   $x = -9y^2 - 16z^2$ opening "backwards"

2. Find the equation of the line containing the points (2, -1, 4) and (5, 0, -3)

$A = (2,-1,4)$ $B = (5,0,3)$

$\vec{AB} = \langle 3, 1, -1 \rangle$

point $\to$ $(2,-1,4)$

line $\vec{r}(t) = \langle 2,-1,4\rangle + t\langle 3,1,-1\rangle$

$x = 2+3t$
$y = -1+t$
$z = 4-t$

3. (a) Find the equation of the plane through (3,1,4) perpendicular to the line $\mathbf{r}(t) = \langle 2, 4, 1\rangle + t\langle -1, 0, 1\rangle$

point: $(3,1,4)$

normal ~~line~~ vector $\langle -1,0,1\rangle$

plane $= -(x-3) + 0(y-1) + 1(z-4) = 0$

$-x+3+z-4=0$

$-x+z = 1$

(b) This plane is parallel to which of the coordinate axes? Explain.

The plane is parallel to the y axis. since it won't intersect

(y-axis is $x = z = 0 \Rightarrow$ no points on the plane)

4. Consider the lines

$$\mathbf{r}(t) = \langle 2, 4, 1\rangle + t\langle 2, -1, 3\rangle$$

and

$$x = 4 - 4t,\ y = 2t - 1\ z = -5 - 6t$$

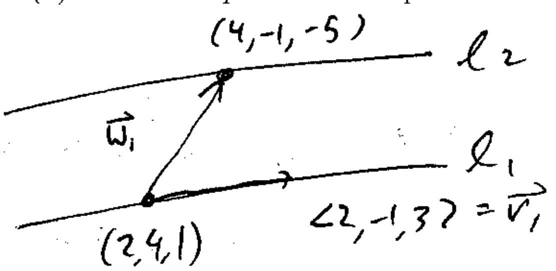
(a) Explain why the lines are parallel.

line 1 : $\vec{v}_1 = \langle 2, -1, 3 \rangle$

line 2 : $\vec{v}_2 = \langle -4, 2, -6 \rangle$. or $\vec{v}_1 = -\frac{1}{2}\vec{v}_2$.

each goes in the same direction.

(b) Find the equation of the plane containing both lines.



$\vec{w}_1 = \langle +2, -5, -6 \rangle$

$\vec{v}_1 \times \vec{w}_1 = \langle 2, -1, 3 \rangle \times \langle +2, -5, -6 \rangle$ $\langle 21, 18, 8 \rangle$

$\vec{K} = \cancel{\langle 21, 6, -12 \rangle}$

plane $= 21(x-4) + 18(y+1) - 8(z+5) = 0$

OR $\cancel{7(x-4) + 2(y+1) - 4(z+5) = 0}$

5. (a) Find the point of intersection of the lines

$$\mathbf{r}_1(t) = \langle 3+t, 3+2t, 2+t \rangle$$

$$\mathbf{r}_2(t) = \langle 3-2t, -t, t-1 \rangle$$

$t \to s$

$3+t = 3-2s \leftarrow t = -2s$

$3+2t = -s$

$2+t = s-1$

$3-4s = -s$

$3 = 3s$

$s = 1 \to t = -2$

And $2+(-2) = 1-1 = 0$ ✓ ✓

So point $= (3-2, 3+2(-2), 2+-2)$

$= (1, -1, 0)$

(b) Find the acute angle of intersection of these two lines. You may leave your answer as an arccosine.

Angle : $\dfrac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|} = \cos\theta$

$\vec{v}_1 \cdot \vec{v}_2 = \langle 1, 2, 1 \rangle \cdot \langle -2, -1, 1 \rangle$

$= -2-2+1 = -3$

need acute, so take $-\vec{v}_1 \cdot \vec{v}_2$ for 3

So $\theta = \arccos \dfrac{3}{\sqrt{6}\sqrt{6}}$

$= \arccos \dfrac{1}{2}$.

6. (Bonus) You may have half a point or one and a half points Extra Credit. Note: If more than 25% of you pick one and a half points, no one gets anything.

I'll be generous. 0.5