KEY

Math 225: Quiz the Fifth

π, 2014

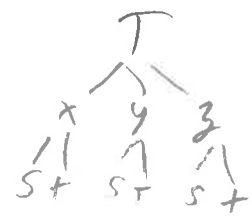
You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e.. no graphing!)

1. Suppose that $T = f(x, y)$. Give a formula for the differential, dT.

$$dT = \frac{\partial f}{\partial x}\, dx + \frac{\partial f}{\partial y} \cdot dy$$

2. Suppose that $T = f(x, y, z)$ and that x, y and z are each functions of s and t. Give a formula for the partial derivative $\dfrac{\partial T}{\partial s}$.

$$\frac{\partial T}{\partial s} = \frac{\partial T}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial T}{\partial z}\frac{\partial z}{\partial s}$$



3. Suppose that $F(x, y, z) = k$ implicitly defines y as a function of x and z. Give a formula for $\dfrac{\partial y}{\partial z}$. State also the conditions under which your formula is valid.

$$\frac{\partial y}{\partial z} = \frac{-F_z}{F_y}$$ provided $F_y \neq 0$.

4. Let $f(x, y) = \ln(2x + y) + \sin(x) + \cos(y)$. Verify Clairaut's Theorem, that is, show that the mixed partials are equal.

$$f_x = \left(\frac{1}{2x+y} \cdot 2\right) + \cos x$$

$$f_{xy} = \frac{-2}{(2x+y)^2} \checkmark$$

$$f_y = \frac{1}{2x+y} - \sin y$$

$$f_{yx} = \frac{-2}{(2x+y)^2} \checkmark$$

5. Suppose that a function $f(x, y)$ has the partial derivative $f_x = x^2 + 2xy$. Give at least three different possibilities for f. (Your answers should differ by more than just a constant).

$$f_x = x^2 + 2xy$$

$$\downarrow$$

$$f = \frac{x^3}{3} + x^2 y + \underline{g(y)}$$

so $f = \frac{x^3}{3} + x^2 y + y^3$

$f = \frac{x^3}{3} + x^2 y + \sin y + \cos y$

$f = \frac{x^3}{3} + x^2 y + \ln(\cos(e^y))$

are all possibilities

6. Let $f(x, y) = 3x^2 - 2xy + y^3$.

(a) Find the equation of the tangent plane to f at the point $(-2, 1)$.

$f(-2,1) = 12 - 2(-2) + 1 = 17$

$f_x(-2,1) = 6x - 2y \Big|_{(-2,1)} = -14$

$f_y(-2,1) = -2x + 3y^2 = 4 + 3 = 7$

$z - 17 = -14(x+2) + 7(y-1)$

(b) Use your tangent plane to approximate $f(-2.02, 1.03)$

$f(-2.02, 1.03) = 17 - 14(-.02) + 7(.03)$

$= 17 + .28 + .21$

$= 17.49$

(c) Find $\nabla(f)$ at the point $(-2, 1)$.

$\vec{\nabla} f = \langle -14, 7 \rangle$ from above

(d) Find $D_{\mathbf{u}}f$ where $\mathbf{u}$ points from $(-2, 1)$ towards the origin.

$\vec{u} = \dfrac{\langle +2, -1 \rangle}{|\langle 2, -1 \rangle|} = \left\langle \dfrac{2}{\sqrt{5}}, \dfrac{-1}{\sqrt{5}} \right\rangle$

$D_u f = \langle -14, 7 \rangle \cdot \left\langle \dfrac{2}{\sqrt{5}}, \dfrac{-1}{\sqrt{5}} \right\rangle = \dfrac{-35}{\sqrt{5}}$

$1 - 2 + 1 + 1$

7. Suppose that $x^2 + 2xyz + xz^2 - y^2z = 1$. Find the tangent plane to this surface at the point $(1, 1, -1)$.

$$\vec{\nabla} F = \langle F_x, F_y, F_z \rangle$$

$$= \langle 2x + 2yz + z^2, 2xz - 2yz, 2xy + 2xz - y^2$$

$$\langle 2 - 2 + 1, -2 + 2, 2 - 2 - 1 \rangle$$

$$= \langle 1, 0, -1 \rangle$$

Tplane: $(x - 1) - (z + 1) = 0$

8. A beachgoer who can't stop thinking about Calc 3 realizes that the temperature function (in degrees Celsius) of the sand is given by $T(x, y) = 100 - 5x^2 - 10y^2$. She is standing at the point (1,2).

(a) How hot are her feet at this point?

$T(1,2) = 100 - 5 - 40 = 55°C$

(b) OUCH! In what direction should she run to cool her feet off the fastest?

Fastest Decrease → direction of $-\vec{\nabla} f$

$$\vec{\nabla} f = \langle -10x, -20y \rangle \Big|_{(1,2)}$$

$$\langle -10, -40 \rangle$$

direction $\left\langle \frac{-1}{\sqrt{17}}, \frac{-4}{\sqrt{17}} \right\rangle \to -\vec{\nabla} f = \left\langle \frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}} \right\rangle$

9. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.) Nov. 19

(b) Of the 44 of us in the two classes, what is the approximate probability that at least 2 of us have the same birthday?

i. 16 % ii. 32 % iii. 70 % iv. 97 %