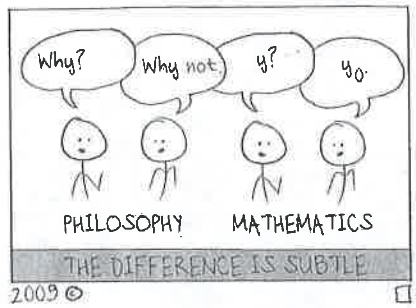
KEY

Math 225: Quiz the Sixth

April 4, 2014

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e., no graphing!)



1. Find and classify the critical points of $f(x, y) = x^5 + y^4 - 5x - 32y - 3$. (There are two of them)

$f_x = 5x^4 - 5 = 0 \Rightarrow 5(x^4 - 1) = 0 \quad x = \pm 1$

$f_y = 4y^3 - 32 = 0 \Rightarrow 4(y^3 - 8) = 0 \quad y = 2$

cp's $(1,2)$ $(-1,2)$

(5)

$D = (20x^3)(12y^2) - (0)^2$

$D_{(1,2)} > 0 \Rightarrow (1,2)$ max or (min) $f_{xx} > 0 \Rightarrow$ min

$D_{(-1,2)} < 0 \Rightarrow (-1,2)$ saddle point

2. Use LaGrange Multipliers to find the maximum value of $f(x,y,z) = x + y - z$ subject to $x^2 + y^2 + z^2 = 2$.

$f = x + y - z$ $\quad g = x^2 + y^2 + z^2 = 2$

$f_x = 1 = 2x\lambda$ $\quad \frac{1}{2\lambda} = x$

$f_y = 1 = 2y\lambda$ $\quad \Rightarrow \frac{1}{2\lambda} = y$

$f_z = -1 = +2z\lambda$ $\quad -\frac{1}{2\lambda} = z$

(4)

$\Rightarrow x = y = -z$

$x^2 + x^2 + (-x)^2 = 2 \Rightarrow 3x^2 = 2 \Rightarrow x = \sqrt{\frac{2}{3}}$ $\quad z = -\sqrt{2/3}$ $\quad \Rightarrow f_{max} = \frac{3\sqrt{2}}{\sqrt{3}}$

$y = \sqrt{2/3}$ $\quad \Rightarrow \sqrt{6}$

3. Find the point on the plane $x + y + z = 1$ closest to $(2, 1, 0)$ (You may use any method that you wish).

$D^2 = (x-2)^2 + (y-1)^2 + z^2$ $\quad$ s.t. $x + y + z = 1$

Lagrange: $2(x-2) = \lambda$

$2(y-1) = \lambda$

$2z = \lambda$

(4)

$\Rightarrow (x-2) = (y-1) = z$

$x = z + 2$ $\quad \Rightarrow (z+2) + (z+1) + z = 1$

$y = z + 1$ $\quad 3z + 3 = 1$

$z = -2/3$ $\quad$ So $(4/3, \frac{1}{3}, -2/3)$

$y = 1/3$

$x = 4/3$

4. Evaluate the integral $\iint_R \cos(x)\sin(y)\, dA$ where $R = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$

$$\int_0^{\pi/2}\int_0^{\pi/2} \cos x \sin y \, dy \, dx$$

$$= \int_0^{\pi/2} \left[-\cos x \cos y\right]_0^{\pi/2} dx$$

(4)

$$= \int_0^{\pi/2} \left[0 - -\cos x\right] dx = \int_0^{\pi/2} \cos x \, dx$$

$$= \sin x \Big]_0^{\pi/2} = 1 - 0 = 1$$

5. Evaluate the integral $\iint_R x^2 + 2xy + y^2 \, dA$ where $R = [0, 4] \times [0, 3]$

$$\int_0^3\int_0^4 x^2 + 2xy + y^2 \, dx \, dy$$

$$= \int_0^3 \frac{x^3}{3} + x^2 y + y^2 x \Big|_0^4 dy$$

(4)

$$= \int_0^3 \frac{64}{3} + 16y + 4y^2 \, dy$$

$$= \frac{64}{3}y + 8y^2 + \frac{4}{3}y^3 \Big]_0^3$$

$$= 64 + 72 + 36 = 172$$

6. Find the volume of the region below $f(x,y) = 2x(x^2+y)^3$ and above the rectangle $R = [1,2] \times [1,3]$

$$\int_1^3 \int_1^2 2x(x^2+y)^3 \, dx \, dy$$

$$u = x^2 + y$$

$$du = 2x \, dx$$

$$\int_1^3 \left[\int_{x=1}^{x=2} u^3 \, du \right] dy$$

$$= \int_1^3 \frac{u^4}{4} \Big]_{x=1}^{x=2} dy = \int_1^3 \frac{(x^2+y)^4}{4} \Big|_{x=1}^{x=2} dy$$

$$\frac{1}{4} \int_1^3 (4+y)^4 - (1+y)^4 \, dy$$

$$= \frac{1}{20} \left[(4+y)^5 - (1+y)^5 \right]_1^3$$

$$= \frac{1}{20} \left[7^5 - 4^5 - 5^5 + 2^5 \right]$$

7. (Bonus) Pretend to toss a coin five times and list the outcomes as a string of Heads and Tails.