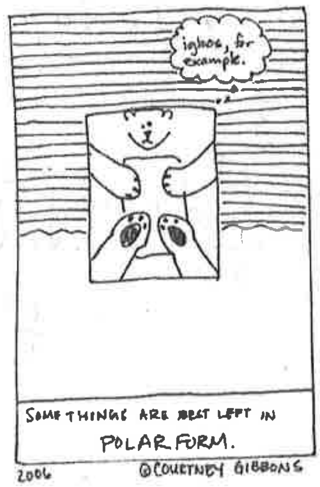
KEY

Math 225: Quiz the Seventh

April 11, 2014

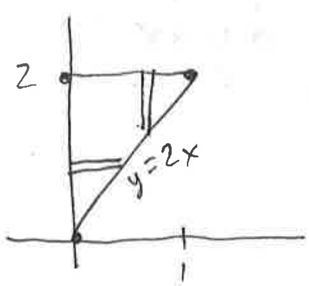
You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e., no graphing!)



1. (a) Set up the bounds for

$$\iint_R x\cos(xy)\,dA$$

where R is the triangle with vertices (0,0), (0,2), and (1,2) twice, using both orders of integration.



a) $\displaystyle\int_0^1 \int_{2x}^2 x\cos xy \; dy\,dx$

or b) $\displaystyle\int_0^2 \int_0^{\frac{1}{2}y} x\cos xy \; dx\,dy$

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(b) If you had to (which you DON'T), which order would you solve and why?

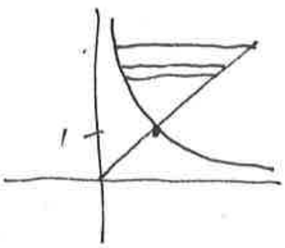
We'd solve (a), as (b) involves an integration by parts, whereas (a) is a straightforward substitution

2. Calculate

$$\iint_R y\,dA$$

where R is bound below by $xy = 1$ and $y = x$ and above by $y = 2$.

$y = \frac{1}{x}$



$$\int_1^2 \int_{\frac{1}{y}}^{y} y\,dx\,dy$$

$$= \int_1^2 yx \Big|_{\frac{1}{y}}^{y} dy = \int_1^2 y^2 - 1\,dy$$

$$= \frac{y^3}{3} - y \Big|_1^2 = \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - 1\right)$$

$$= \boxed{\frac{4}{3}}$$

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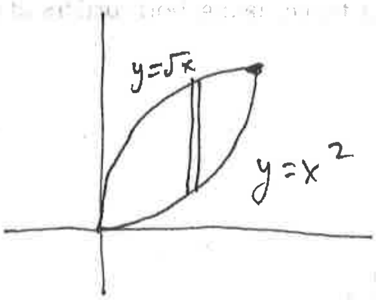
3. Set up the integral to calculate the volume below $z = x^2 - 2x + y^2 + 3$ and above the region in the first quadrant bound by $y = x^2$ and $y = \sqrt{x}$.



$$\int_0^1 \int_{x^2}^{\sqrt{x}} x^2 - 2x + y^2 + 3\ dy\,dx$$

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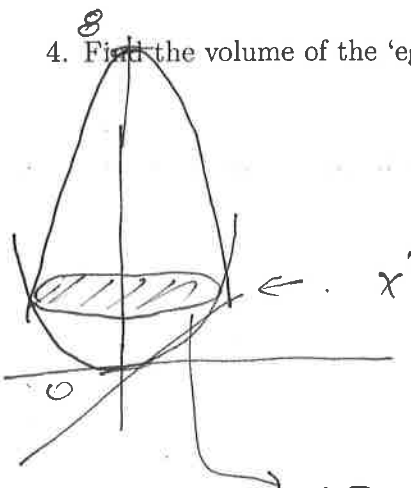
4. Find the volume of the 'egg' bound by the paraboloids $z = x^2 + y^2$ and $z = 8 - 3x^2 - 3y^2$.



$$\iint_D (8-3x^2-3y^2)-(x^2+y^2)\,dA$$

$x^2+y^2 = 8-3x^2-3y^2$

$4x^2+4y^2 = 8$

$x^2+y^2 = 2$

$0 \le r \le \sqrt{2}$

$0 \le \theta \le 2\pi$

$$\int_0^{2\pi}\int_0^{\sqrt{2}} (8-4r^2)r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} 8r-4r^3\,dr\,d\theta$$

$$= \int_0^{2\pi}\left[4r^2-r^4\right]_0^{\sqrt{2}}\,d\theta$$

$$= \int_0^{2\pi} 8-4\,d\theta = \underline{8\pi}$$

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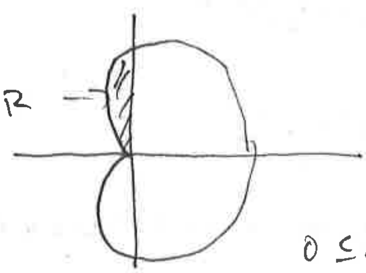
5. Calculate the area in the second quadrant bound by the cardioid $r = 1 + \cos(\theta)$.

R



$$\iint_R 1\,dA = \int_{\frac{\pi}{2}}^{\pi}\int_0^{1+\cos\theta} r\,dr\,d\theta$$

$0 \le r \le 1+\cos\theta$

$\frac{\pi}{2} \le \theta \le \pi$

$$= \int_{\pi/2}^{\pi} \frac{(1+\cos\theta)^2}{2}\,d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{2}}^{\pi} 1 + 2\cos\theta + \cos^2\theta\,d\theta$$

$\hookrightarrow \frac{1+\cos 2\theta}{2}$

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$$= \frac{1}{2}\left[\theta + 2\sin\theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi}$$

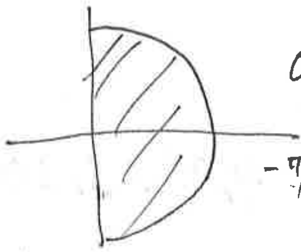
$$= \frac{1}{2}\left[\frac{3\theta}{2} + 2\sin\theta + \frac{\sin 2\theta}{4}\right]_{\pi/2}^{\pi} = \frac{1}{2}\left(\frac{3\pi}{2}+0+0-\left(\frac{3\pi}{4}+2+0\right)\right]$$

$$= \frac{1}{2}\left(\frac{3\pi}{4}-2\right) = \frac{3\pi}{8}-1$$

6. Convert the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{x^2+y^2}\, dy\, dx$$

to polar coordinates. You need not compute the integral. Also, give a rough sketch of the volume being computed by this integral.



$0 \le r \le 1$

$-\pi/2 \le \theta \le \pi/2$

$$\int_{-\frac{\pi}{2}}^{\pi/2} \int_0^1 r \cdot r \, dr\, d\theta$$

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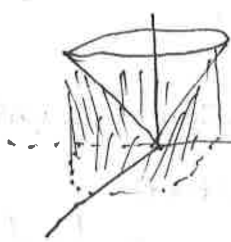
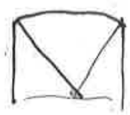


← cone

← volume beneath half of the cone



← "front" view.

7. (Bonus) Tell me about an interesting talk or poster that you saw at the Undergraduate Conference on Tuesday.

Great work on Tuesday. I saw most of you at @ least one presentation or poster, ~~so~~ Loved the enthusiasm!