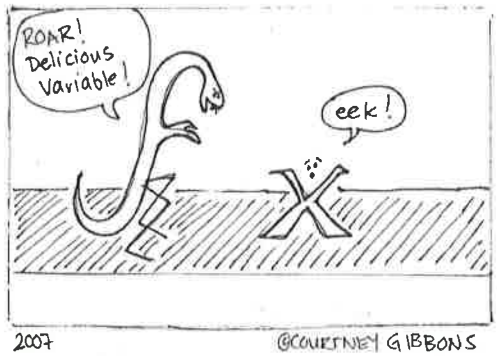
KEY

Math 225: Quiz the Eighth

May 2, 2014

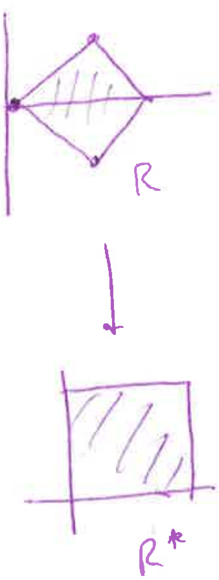
You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e., no graphing!)



1. Calculate

$$\iint_R (x+y)^2 e^{x-y}\, dA$$

where R is the square with vertices $(0,0), (1,1), (2,0)$ and $(1,-1)$. I'll 'sell' you the substitution if you can't figure it out.



$u = x+y$

$v = x-y$

$x = \frac{1}{2}u + \frac{1}{2}v$

$y = \frac{1}{2}u - \frac{1}{2}v$

$$\mathcal{J} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$|\mathcal{J}|$

$$\iint_R (x+y)^2 e^{x-y}\, dA = \frac{1}{2}\int_0^2 \int_0^2 u^2 e^v \, du\, dv$$

$$= \frac{1}{2}\int_0^2 \frac{u^3}{3} e^v \Big|_0^2 \, dv$$

$$= \frac{1}{2}\int_0^2 \frac{8}{3} e^v \, dv$$

$$= \frac{4}{3}\left(e^2 - 1\right)$$

2. Find $\iiint_E z\, dV$ where E is the region bound above by the sphere of radius 2 and below by the cone $z = \sqrt{x^2+y^2}$. Do this using

 ← intersection: circle of radius $\sqrt{2}$.

(a) Cylindrical Coordinates

$$\int_0^{2\pi}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} z\, r\, dz\, dr\, d\theta = \int_0^{2\pi}\int_0^{\sqrt{2}} \frac{z^2}{2} r \Big|_r^{\sqrt{4-r^2}} dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} 2r - \frac{r^3}{2} - \frac{r^3}{2}\, dr\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\sqrt{2}} 2r - r^3\, dr\, d\theta$$

$$= \int_0^{2\pi} r^2 - \frac{r^4}{4}\Big|_0^{\sqrt{2}} d\theta \qquad \int_0^{2\pi} 2-1\, d\theta = \underline{2\pi}$$

(b) Spherical Coordinates

Bonus term.

$$\int_0^{\frac{\pi}{4}}\int_0^{2\pi}\int_0^{2} \underbrace{\rho\cos\phi}_{z}\ \underbrace{\rho^2\sin\phi}_{d}\, d\rho\, d\theta\, d\phi$$

$$= \int_0^{\frac{\pi}{4}}\int_0^{2\pi} \frac{\rho^4}{4}\cos\phi\sin\phi\Big|_0^2 d\theta\, d\phi$$

$$= \int_0^{\frac{\pi}{4}}\int_0^{2\pi} 4\cos\phi\sin\phi\, d\theta\, d\phi = \int_0^{\frac{\pi}{4}} 8\pi\cos\phi\sin\phi\, d\phi = 4\pi\sin^2\phi\Big|_0^{\pi/4}$$

$$= 4\pi\left(\frac{\sqrt{2}}{2}\right)^2 = \underline{2\pi}$$

3. Find $\int_C xy\, ds$ where C is the line segment from $(1,2)$ to $(4,6)$.

$$\int_C xy\, ds = \int_0^1 (1+3t)(2+4t)\cdot 5\, dt$$

C: $x(t) = 1+3t$

$y(t) = 2+4t$

$ds = \sqrt{3^2+4^2} = 5$

$0 \le t \le 1$

$$= 5\int_0^1 2 + 10t + 12t^2\, dt$$

$$= 5\left[2t + 5t^2 + 4t^3\right]_0^1$$

$$= 5(11) = 55.$$

4. Find the work done moving an object from $(1,1)$ to $(3,9)$ along the parabolic segment $y = x^2$ where the force acting on the object is given by $\langle x\cos y, xy\rangle$.

$$\text{Work} = \int_C \vec{F}\cdot d\vec{r} = \int_C P\,dx + Q\,dy$$

C: $x(t) = t$

$y(t) = t^2$

$(1 \le t \le 3)$

$$= \int_C x\cos y\, dx + xy\, dy$$

$$= \int_1^3 (t\cos t^2)(1) + t^3(2t)\, dt$$

$$= \frac{\sin t^2}{2} + \frac{2t^5}{5}\Big|_1^3$$

$$= \frac{\sin 9}{2} + \frac{486}{5} - \frac{\sin 1}{2} - \frac{2}{5}.$$

Bonus: Write a caption for the cartoon on the first page.