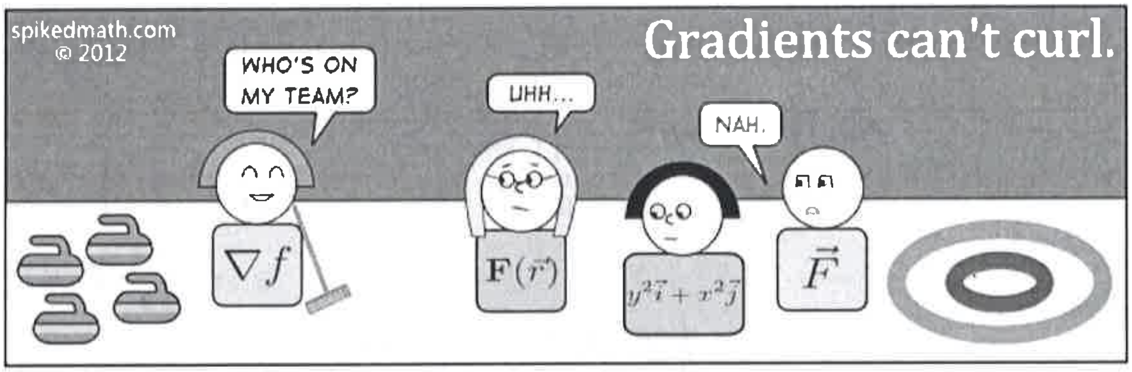
KEY

Math 225: Quiz the Eighth
May 2, 2014

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e., no graphing!)



1. Find

$$\int \underbrace{3x^2 + 2xy}_{P}\, dx + \underbrace{x^2 - y}_{Q}\, dy$$

where C is the line segment from (1,1) to (2,6).

$\frac{\partial Q}{\partial x} = 2x \qquad \frac{\partial P}{\partial y} = 2x \Rightarrow \vec{F}$ is conservative

$f_x = 3x^2 + 2xy \qquad f_y = x^2 - y \qquad g'(y) = -y$

$f = x^3 + x^2 y + g(y) \to f_y = x^2 + g'(y) \qquad g(y) = -\frac{y^2}{2}$

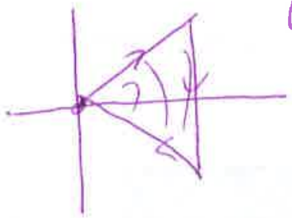
$f = x^3 + x^2 y - \frac{y^2}{2}$

FTLI: $\int_C \vec{F}\cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a)) = x^3 + x^2 y - \frac{y^2}{2} \Big|_{(1,1)}^{(2,6)}$

$= (8 + 24 - 18) - (1 + 1 - \frac{1}{2})$

$= \underline{12.5}$

2. Find $\int_C (x^2 + y^2)\, dx + (x + y^2)\, dy$, where C consists of the line segments from (0,0) to (1,1) to (1, -1) back to (0,0).



C closed $\qquad \frac{\partial Q}{\partial x} = 1 \qquad \frac{\partial P}{\partial y} = 2y \quad \Rightarrow \vec{F}$ is not conservative

Greens Thm, but note C runs clockwise.

Thus

$$\oint_C P\,dx + Q\,dy = -\iint_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA = -\int_0^1 \int_{-x}^{x} 1 - 2y\, dy\, dx$$

$$= -\int_0^1 y - y^2 \Big|_{-x}^{x} dx$$

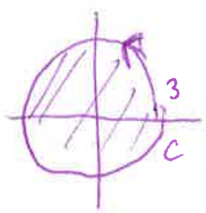
$$= -\int_0^1 2x\,dx = \cancel{\int_0^1 2x\,dx}$$

$$= -x^2 \Big]_0^1 = \boxed{-1}$$

3. Find

$$\oint_C x^2 + 2y\; dx + 4x + y^4\; dy$$

over the circle of radius 3, centered at the origin, oriented counterclockwise.



Greens Thm

$$\int_C (x^2 + 2y)\,dx + (4x + y^4)\,dy$$

$$= \iint_R 4 - 2\; dA = 2 \iint_R dA = 2\ \text{Area}(R)$$

$$= 2(9\pi) = \boxed{18\pi}$$

4. Find the work done by the force field $\mathbf{F} = \langle yz, xz, xy + z^2 \rangle$ in moving an object from (1,1,1) to (2,8,4) along the path $\mathbf{r}(t) = \langle t, t^3, t^2 \rangle$

$$\vec{F} = \langle yz, xz, xy + z^2 \rangle = \vec{\nabla}\left(xyz + \frac{z^3}{3}\right)$$

can check curl $= \vec{0}$, or that the derivatives work out.

$$\text{so Work} = \int_C \vec{F} \cdot d\vec{r} = xyz + \frac{z^3}{3} \Big|_{(1,1,1)}^{(2,8,4)}$$

$$= 64 + \frac{64}{3} - 1 - \frac{1}{3} = 63 + \frac{63}{3} = \boxed{84}$$

5. Consider the vector field $\mathbf{F} = \langle xz, e^{xy}, 2y^3 \rangle$

(a) Find curl($\mathbf{F}$)

$$\text{curl}(\vec{F}) = \vec{\nabla} \times \vec{F}$$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & e^{xy} & 2y^3 \end{vmatrix} = \langle 6y^2 - 0, x - 0, ye^{xy} - 0 \rangle$$

$$= \langle 6y^2, x, ye^{xy} \rangle$$

(b) Find div($\mathbf{F}$)

$$\text{div}(\vec{F}) = \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(e^{xy}) + \frac{\partial}{\partial z}(2y^3)$$

$$= z + xe^{xy}$$

(c) Explain why one statement below is nonsense. Verify the other statement in the case of the field above.

$div(curl(\mathbf{F})) = 0$ ← sense

$curl(div(\mathbf{F})) = \mathbf{0}$ ← nonsense, div($\vec{F}$) is a scalar, cant curl a scalar.

$$\text{div}(\text{curl}(\vec{F})) = \frac{\partial}{\partial x}(6y^2) + \frac{\partial}{\partial y}(x) + \frac{\partial}{\partial z}(ye^{xy}) = 0 + 0 + 0 = 0$$ ✓

6. (Bonus) What concepts are you feeling most confident about going into the final? What are you most 'worried' about?