KEY

Math 225: Quiz Zero

These problems are representative of the type that you have encountered in your mathematical past, and give some of the concepts that you'll be responsible for as we move through the new material. This will not be collected, though I will provide a solutions key online.

1. Give the domain and range for the function $f(x) = \ln(1 - x^2)$.

ln is defined for positive values, thus we need

$1-x^2>0$ or $1>x^2$, which gives $-1<x<1$

Range: $1-x^2>0$ and $1-x^2<1$ ⇐ (since $x^2 \geq 0$)

so $-\infty < f(x) \leq 0$

2. Find

$$\lim_{x\to 2}\frac{x^2-3x+2}{x^2-5x+6} = \lim_{x\to 2}\frac{(x-1)(x-2)}{(x-2)(x-3)}$$

$$= \lim_{x\to 2}\frac{(x-1)}{(x-3)} = \frac{1}{-1} = -1$$

3. Find the maximum and minimum values of $f(x) = 3x - x^2$ on the interval $[1, 4]$

Crit points: $f'(x)=0 \to 3-2x=0 \Rightarrow x = 3/2$ $\quad f(3/2) = \frac{9}{2} - \frac{9}{4} = \frac{9}{4}$

endpoints $f(1) = 3-1 = 2$

$f(4) = 12-16 = -4$

MAX: $9/4$ MIN: -4

4. Let $f(x) = \sqrt[3]{x}$. Find the tangent line to $f(x)$ at $x = 8$ and use it to approximate $\sqrt[3]{8.02}$.

$f(8) = 2$

$f(x) = x^{1/3}$

$f'(x) = \frac{1}{3}x^{-2/3}$

$f'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$

T line $y-2 = \frac{1}{12}(x-8)$

$f(8.02) \approx 2 + \frac{1}{12}(8.02-8)$

$= 2 + \frac{.02}{12} = 2.0016666\ldots$

(NB: $\sqrt[3]{8.02} = 2.00166528\ldots$)

5. (a) Find

$$\int_0^{0.5} \frac{t\,dt}{\sqrt{1-t^4}}$$

$u = t^2$ $ll: 0$

$du = 2t\,dt$ $ul = \frac{1}{4}$

$$\frac{1}{2}\int_0^{\frac{1}{4}} \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} \arcsin u \Big|_0^{1/4} = \frac{1}{2} \arcsin \frac{1}{4}.$$

(b) Discuss

$$\int_0^1 \frac{t\,dt}{\sqrt{1-t^4}}$$

Our function is undefined @ $t=1$, so the integral is improper!

We need to do $\lim_{b \to 1^-} \int_0^b \frac{t\,dt}{\sqrt{1-t^4}} = \lim_{b \to 1} \frac{1}{2} \arcsin t^2 \Big]_0^b = \frac{1}{2}(\arcsin(1) - \arcsin 0)$

$= \frac{\pi}{4}$

6. Find

$$\int \cos^2\theta\, d\theta \text{ and } \int \cos^3\theta\, d\theta$$

$\int \cos^2\theta\, d\theta = \int \left[\frac{1+\cos 2\theta}{2}\right] d\theta$

$= \frac{\theta}{2} + \frac{\sin 2\theta}{4} + C$

$\int \cos^3\theta\, d\theta = \int \cos^2\theta\,(\cos\theta)\,d\theta$

$= \int (1-\sin^2\theta)\cos\theta\, d\theta$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$= \int 1-u^2\,du = u - \frac{u^3}{3} + C = \sin\theta + \frac{\sin^3\theta}{3} + C$

7. Find both $\frac{d}{dx}xe^x$ and $\int xe^x\,dx$

$\frac{d}{dx}(x e^x) = \frac{d}{dx}(x)\cdot e^x + \frac{d}{dx}(e^x)\cdot x$

$= e^x + x e^x.$

$\int xe^x dx$ BY PARTS!

$u = x$ $dv = e^x dx$

$du = dx$ $v = e^x$

$uv - \int v\,du$

$xe^x - \int e^x dx$

$= xe^x - x + C$