Math 225: Practice Exam the First

This exam is meant to cover the types of problems that you might encounter on Friday's Midterm. It is by no means exhaustive. To get the most out of your studies, you should also review our past quizzes, homeworks, and class notes. Come prepared with questions on this for Wednesday.

1. Consider the parametric curve given by $x = -2t, y = 8t - 2t^2$
   (a) Find the slope of the tangent line when $t = 1$.
   (b) Find the area bound by this curve and the x-axis. Be sure to explain any 'adjustments' you might need to make in your calculation.
2. Explain the similarities and differences between the curves $\mathbf{r}(t) = \langle t^2, 2t^2 + 1\rangle$ and $\mathbf{r}(t) = \langle \frac{t-1}{2}, t\rangle$.
3. Consider the points $A = (3, 1, 2)$, $B = (7, -1, 4)$ and $P = (x, y, z)$.
   (a) Find the distance from A to B.
   (b) Write the following as a mathematical equation involving a dot product.
   $\overrightarrow{AP} \perp \overrightarrow{BP}$
   (c) Simplify your equation into the form of a sphere and identify its center and radius.
   (d) What relationships are there between the center, radius, and original points A and B?
4. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors. Prove that if $|\mathbf{a} + \mathbf{b}| = |\mathbf{a} - \mathbf{b}|$, then $\mathbf{a}$ and $\mathbf{b}$ are orthogonal.
5. Suppose that we have three lines, each of which is skew to the other two. Might there be a plane parallel to all 3 lines? Must there be a plane parallel to all 3 lines? Explain.
6. Let $\mathbf{a} = \langle 1, 3, 4\rangle$ and $\mathbf{b} = \langle < 1, 0, -3 >$. Find vectors $\mathbf{x}$ and $\mathbf{y}$ such that $\mathbf{x}$ is parallel to $\mathbf{a}$, $\mathbf{y}$ is perpendicular to $\mathbf{a}$ and $\mathbf{b} = \mathbf{x} + \mathbf{y}$.
7. Consider the points $A = (2, -1, 5)$, $B = (4, -3, 1)$, and $C = (1, -2, 3)$.
   (a) Find the equation of the line through A and B.
   (b) Find $\angle BCA$. You may leave your answer as an arccosine.
   (c) Find the equation of the plane containing $\triangle ABC$.
   (d) Find the area of $\triangle ABC$.
8. (a) Identify the following two surfaces. Draw a rough sketch of each (I will be generous on grading these. Make sure things point in the right directions):
   $$y^2 + z^2 + 1 = x^2$$
   $$2z = x^2 - y^2$$
   (b) Find the intersection of the two surfaces. Show that they intersect in a plane. (You need not parametrize the intersection).

9. (a) Find the tangent line to $\mathbf{r}(t) = \langle t^2 + 1, t^3 + t, 3t + 1 \rangle$ at the point when $t = 2$.
   (b) Find the speed of $\mathbf{r}(t)$ when $t = 0$.

10. (a) Determine if the following lines are parallel, intersect, or are skew.

$$\mathbf{r}_1(t) = \langle 1, -1, 0 \rangle + t\langle 1, 4, 2 \rangle$$
$$\mathbf{r}_2(t) = \langle 2 - t, 3 + t, 1 + 3t \rangle$$

   (b) If they are parallel or intersect, find the plane containing both of them. If they are skew, find a plane parallel to both of them through the origin.