Math 225: Practice Exam the Second

This problem set is taken from last year's second midterm. It is representative of the types of problems you will see, but is by no means exhaustive.

1. Let $f(x,y) = \frac{1}{\sqrt{x^2+y^2-1}}$
   (a) What is the domain of f? Give both an algebraic description and a sketch.
   (b) Draw level curves for f for values of $\frac{1}{2}, 1, 2$
   (c) Are there level curves for $f \leq 0$? Explain.
2. Let $f(x,y) = x^2 + y^3 + 4y + x\cos(y)$
   (a) Find the tangent plane at the point $(3,0)$ and use it to approximate $f(\pi, 0.1)$
   (b) Find $D_{\mathbf{u}}f$ at $(3,0)$ as we move towards $(2,1)$.
3. (a) Find the equation to the tangent plane to the surface $x^3y + y^3z + z^3x = 5$ at the point (2,1,-1).
   (b) Find $\dfrac{\partial z}{\partial y}$ at this point.
4. Let $f(x,y) = x^2 - 2x + 2y^2 - 4y + 5$.
   (a) Find and classify the only critical point of f.
   (b) Find the maximum and minimum value of f on the triangle with vertices $(0,0), (2,4)$ and $(2,0)$. (The diagonal line bound is computationally messy. See me if you need a calculator).
5. Calculate
$$\int_0^3 \int_y^3 e^{x^2}\, dx\, dy$$
   by first reversing the order of integration.
6. Find the volume bound by the paraboloid $z = 2 - x^2 - y^2$ and $z = 1$.
7. Find the volume of the unit sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant using *calculus* (You may check your answer if you remember the formula for the volume of a sphere!).
8. Set up, but don't compute, the integral for the surface area of $f(x,y) = \sqrt{x^2+y^2-1}$ over $[3,4] \times [4,6]$.
9. Find the center of mass of a plate in the shape of the area between the curves $y = x^2$ and $x = y^2$ in the first octant, if the density of the plate is given by the function $\rho(x,y) = xy$. (Hints: You may save yourself some work by employing symmetry. The mass of this plate is $\frac{1}{12}$).