Math 225: Final Review

Spring 2014

We will review these on Tuesday evening.

1. Let $\mathbf{a} = \langle 1, 2, 4\rangle$, $\mathbf{b} = \langle 0, 3, 6\rangle$ and $\mathbf{r} = \langle x, y, z\rangle$. Show that the set of $\mathbf{r}$ such that $(\mathbf{r}-\mathbf{a})\cdot(\mathbf{r}-\mathbf{b}) = 0$ is a sphere, and find its center and radius (for a bonus, you may comment on its center and radius).

2. (a) Find the plane containing the points (0,0,2), (1,2,3) and (-1,4,5).

   (b) Does the line $\ell(t) = \langle 1 + 3t, 5, 2 - t\rangle$ intersect this plane? *Where* or why not?

3. Let $\mathbf{r}(t) = \langle \cos(t), \sin(t), \cos^2(t)\rangle$.

   (a) What range of values does this curve take on in each component?

   (b) Find (and identify) two surfaces whose intersection is this curve.

   (c) Find the tangent line to this curve when $t = \dfrac{\pi}{4}$

4. Let $f(x, y) = xy + x^2 - e^y$.

   (a) Find the tangent plane to $f(x, y)$ at the point $(3, 0)$.

   (b) Find $D_{\mathbf{u}}f$ as we move from $(3, 0)$ to $(4, 2)$.

5. Find the minimum value of $f(x, y, z) = 3x + 2y + z$ if x, y and z are positive numbers such that $xyz = 100$

6. Find $\iint_R x + y\, dA$ where R is bound by $x = 0$, $y = 4$, and $y = x^2$. Do this twice, once using each order of integration, and verify that your answers are the same.

7. Find the area contained in *both* of the curves $x^2 + y^2 = 2x$ and $x^2 + y^2 = 2y$. (Hint: Convert both equations to polar coordinates).

8. Evaluate
$$\iiint_E (9 - x^2 - y^2)\, dV$$
   where H is the upper hemisphere of radius 3.

9. Find $\int_C 2xy\, dx + x^2 + y^2\, dy$ where C is the line segment from $(1, 1)$ to $(4, 5)$.

10. Find $\oint_C 3y\, dx + x^2\, dy$ where C is the square from $(0, 0)$ to $(1, 0)$ to $(1, 1)$ to $(0, 1)$ and back to the origin.

11. Find $\oint_C x^2\, dx + \cos(y)\, dy + e^z\, dz$, where C is the curve of intersection of $x^2 + y^2 = 1$ and $z = x^2 - y^2$.

12. Find the set of points equidistant from $A = (2, 3, 4)$ and $B = (6, -1, 10)$. That is, find the points $P = (x, y, z)$ such that $d(P, A) = d(P, B)$. What common surface is this?

13. Let $\mathbf{u}$ and $\mathbf{v}$ be unit vectors in $\mathbb{R}^3$.

    (a) What is the maximum value of $|\mathbf{u} \times \mathbf{v}|$ and when (geometrically) does this occur?

    (b) What is the minimum value of $|\mathbf{u} \times \mathbf{v}|$ and when (geometrically) does this occur? (two answers here).

14. Let $\mathbf{r}(t) = \langle t, 2t, t^3 \rangle$.

    (a) Find the tangent line to $\mathbf{r}(t)$ when $t = 1$.

    (b) Find the speed and the acceleration of $\mathbf{r}(t)$ when $t = 1$.

15. Let

$$f(x, y) = x^2 + 2xy + \ln(y)$$

    (a) Find the tangent plane to $f(x, y)$ at $(2, 1)$

    (b) Find the derivative at $(2, 1)$ as we move towards the origin.

16. Find and classify the critical points of $f(x, y) = 3x - x^3 - 2y^2 + y^4$. (There are six of them).

17. (a) Find the area of the triangle with vertices $(0, 0)$, $(a, 0)$ and $(0, b)$ using calculus. (You'll need to find the line with the last two points...)

    (b) Find the *volume* of the *tetrahedron* with vertices $(0, 0, 0)$, $(a, 0, 0)$, $(0, b, 0)$, and $(0, 0, c)$. (Verify here that the plane with the last three points has equation $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$).

    (c) If there were a four dimensional analog to this problem with a four-dimensional figure, what might it's volume be?

18. Find

$$\int_C (y \cos(x))\, dx + (y + \sin(x))\, dx$$

where C is any path from $(0, 1)$ to $(\frac{\pi}{2}, 1)$.

19. Find

$$\oint_C (x^2 + 2y)\, dx + (3x + \sqrt{y})\, dy$$

where C starts at (1,0), traverses the top half of the unit circle, counterclockwise, then returns to the start point along the x-axis.

20. Let $z^2 = 1 + x^2 + y^2$.

    (a) What quadric surface is the graph of this equation?

    (b) Find the volume in the first octant bound by this surface and the plane $z = 3$.

These two questions WILL be on your final exam

21. State carefully as many (at least 3) integral theorems as you can and draw any parallels that you can between them. Note: I want *theorems* and not *formulas* (ie, 'The integral for arclength' is not a *theorem*).

22. Choose a concept from single-variable calculus that was revisited in multivariable calculus and explain the parallels and the differences between the concept in the two contexts. Where applicable, be sure to give *geometric, algebraic, and numeric* explanations of your concept. Write your answer to someone who is finishing Calculus II and is thinking about taking Calculus III.