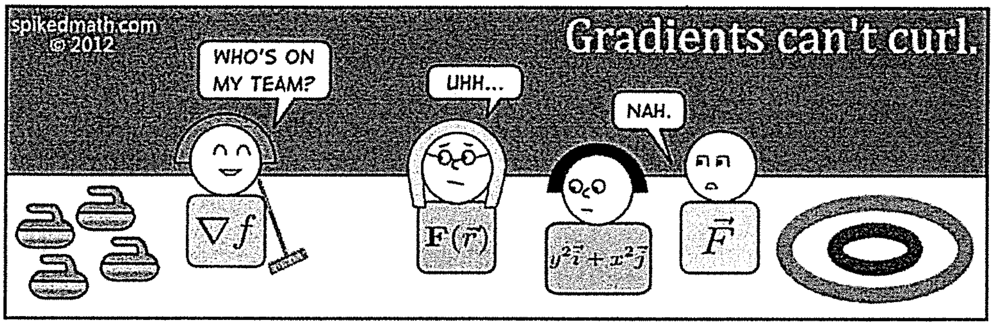
KEY

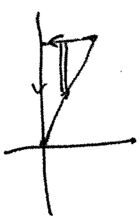
Math 225: Quiz the Last

May 8, 2015

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find $\oint_C 2x + y\ dx + 2xy\ dy$ where C is the triangle with vertices $(0,0)$, $(3,6)$ and $(0,6)$.



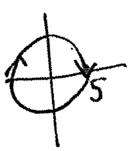
$$\oint_{\partial D} P\,dx + Q\,dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA = \int_0^3 \int_{2x}^6 2y - 1\ dy\,dx$$

$$= \int_0^3 y^2 - y \Big]_{2x}^6 dx = \int_0^3 30 - 4x^2 + 2x\,dx$$

$$= \left[30x - \frac{4}{3}x^3 + x^2 \right]_0^3$$

$$= 63$$

2. Find $\oint_C (e^x + x^2y)\ dx + (\ln(2y^2+1) - xy^2)\ dy$ where C is the circle of radius 5, centered at the origin, traversed clockwise.



For clockwise

$$\oint_{\partial D} P\,dx + Q\,dy = \iint_D \frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\,dA = \iint_D x^2 + y^2\,dA$$

$$= \int_0^{2\pi} \int_0^5 r^2 \cdot r\,dr\,d\theta$$

$$= \frac{5^4}{4} \cdot 2\pi$$

3. Let $\mathbf{F} = \langle xyz^2, xy^2z, x^2yz \rangle$

(a) Find **curl**(**F**).

$$\begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xyz^2 & xy^2z & x^2yz \end{vmatrix} = \langle x^2z - xy^2, 2xyz - 2xyz, y^2z - xz^2 \rangle$$

$$= \langle x^2z - xy^2, 0, y^2z - xz^2 \rangle$$

(b) Find *div*(**F**).

$$\frac{\partial (xyz^2)}{\partial x} + \frac{\partial (xy^2z)}{\partial y} + \frac{\partial (x^2yz)}{\partial z} = yz^2 + 2xyz + x^2y$$

(c) Verify that *div*(**curl**(**F**)) = 0.

$$div(curl(\vec{F})) = \frac{\partial}{\partial x}(x^2z - xy^2) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(y^2z - xz^2)$$

$$= 2xz - y^2 + y^2 - 2xz = 0$$

4. Let $\mathbf{F} = \langle 2xy, x^2 + z^2, 2xz \rangle$. Is there a vector field **G** such that **F** = **curl**(**G**)? Why or why not?

$div(curl(\vec{G})) = 0$

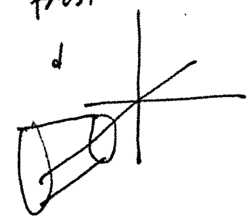
but $div(\langle 2xy, x^2 + z^2, 2xz \rangle)$

$= 2y + 0 + 2x \neq 0$

so no field $\vec{G}$ exists.

5. Find parametric equations (with bounds!) for the portion of the cone $4x^2 = y^2 + z^2$, between the planes $x = 2$ and $x = 7$. (As a bonus, what is the name of this shape?)

frustrum!



$y = r\cos\theta \qquad 0 \le \theta \le 2\pi$

$z = r\sin\theta \qquad 4 \le r \le 14$

$x = \frac{r}{2}$

$4x^2 = 4\left(\frac{r}{2}\right)^2 = r^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$= y^2 + z^2$

6. Find the surface area of $z = xy$ that lies inside of the cylinder $x^2 + y^2 = 1$

↳ surface ↳ bounds

$x = u \qquad \vec{r}_u = \langle 1, 0, v \rangle$

$y = v \qquad \vec{r}_v = \langle 0, 1, u \rangle$

$z = uv$

$d\vec{S} = \langle -v, -u, 1 \rangle \, du\,dv$

$dS = \sqrt{v^2 + u^2 + 1} \; du\,dv$

$S.A. = \iint \sqrt{v^2 + u^2 + 1} \, du\,dv$

$= \int_0^{2\pi} \int_0^1 \sqrt{r^2 + 1} \, r \, dr\, d\theta$

$\int_0^{2\pi} \frac{1}{3}(r^2 + 1)^{3/2} \Big|_0^1 d\theta$

$= 2\pi\left(\frac{2\sqrt{2} - 1}{3}\right)$

7. (Bonus) Let $f(x, y, z)$ be any function of three variables. Is there a vector field $\mathbf{F}$ such that $f(x, y, z) = div(\mathbf{F})$? If so, explain how to construct one.

yes. Let $g(x, y, z) = \int_0^t f(t, y, z)\, dt$

↑ antideriv of f w.r.t. x.

Then If $\vec{F} = \langle g(x,y,z), 0, 0 \rangle$

Then $div(\vec{F}) = f(x,y,z)$