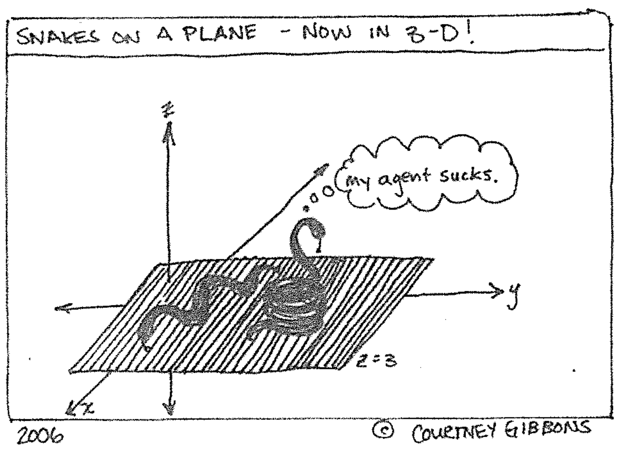
KEY

Math 225: Quiz the First
January 30, 2015

This quiz is closed book and closed notes. You may use a calculator for arithmetic only, that is, no graphing and no calculus. Please justify all of your answers. You have until the end of the class period to finish.



1. Describe the similarities and differences between the parametric equations given by

$$x = t, y = t + 1$$

$$x = t^2 - 1, y = t^2$$

$$x = \sin(t), y = \sin(t) + 1$$

each for all real t.

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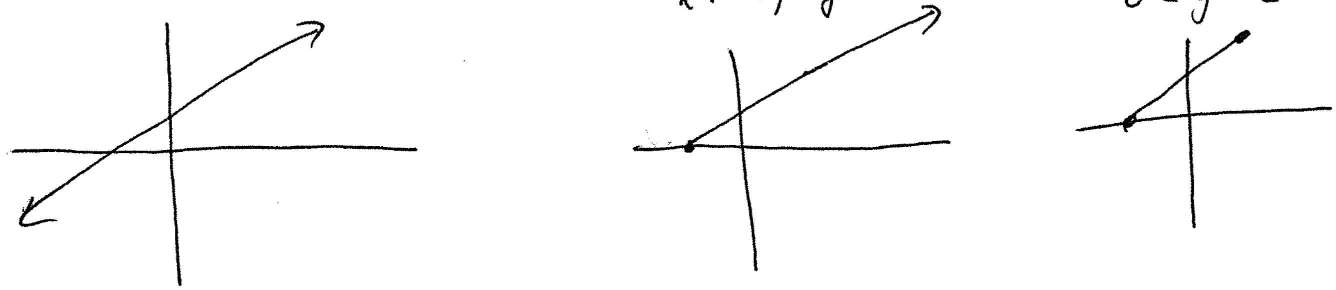
→ Each is a variant on the line $y = x + 1$

The first is the whole line

The second is all $x > -1$, $y > 0$

The third is $-1 < x < 1$, $0 < y < 2$



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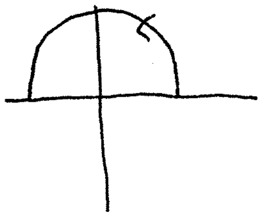
2. (a) Find parametric equations (and t values!) for the upper half unit circle, traversed counter-clockwise, starting at (1,0)

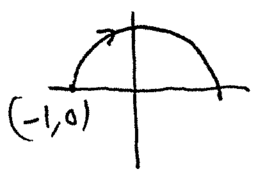


$x = \cos t$

$y = \sin t$

$0 \le t \le \pi$

(b) Find parametric equations (and t values!) for the upper half circle, traversed clockwise, starting
at (-1, 0)



$x = -\cos t$ $\quad 0 \le t \le \pi$.

$y = \sin t$

(c) Use calculus to find the length of this upper half circle. (You may check your answer with high-school geometry).

$$\int_0^\pi \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^\pi \sqrt{(-\sin t)^2 + (\cos t)^2}\, dt$$

$$= \int_0^\pi 1\, dt = \underline{\pi}$$

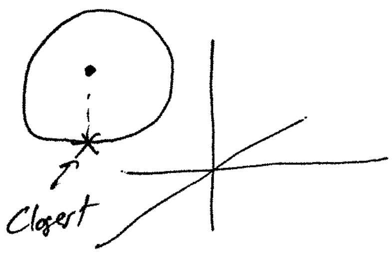
$$\left(= \frac{1}{2}(2\pi R) \right)$$

circumference

4. (a) Find the equation of the sphere with center $(4, -2, 6)$ and radius 3.

Sphere: $(x-4)^2 + (y+2)^2 + (z-6)^2 = 9$

(b) Which point on this sphere is closest to the xy-plane?



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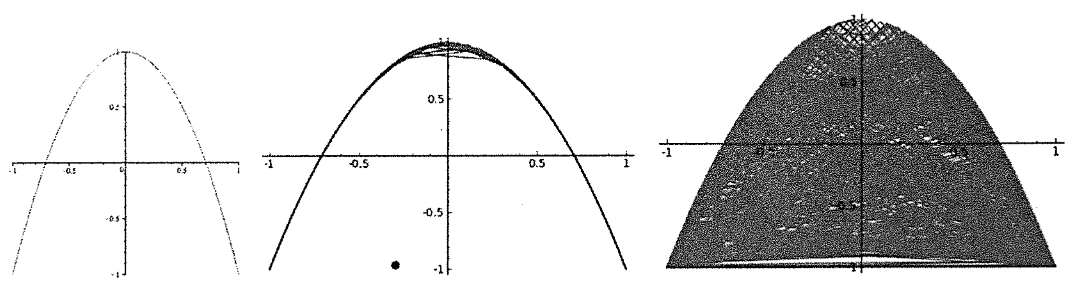
$(4, -2, 3)$

(c) Find the distance between the center and the point $(2, 2, 10)$.

Distance: $(4,-2,6)$ and $(2,2,10)$

$= \sqrt{(4-2)^2 + (-2-2)^2 + (6-10)^2} = \sqrt{4+16+16} = 6.$

5. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t)$, which we showed in class to be a parabola similar to $y = 1 - 2x^2$, but restricted and periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 2\pi)$, $(0, 100\pi)$, and $(0, 500\pi)$ The 'dot' in the second graph is merely a printing error).

The computer draws by plotting points and connecting lines. The greater the range, the further apart the lines, the more the inaccuracy.

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3. Consider the curve $x = e^{-t}$ and $y = 1 - t^2$, for all real values t.

(a) Which values do x and y take on?

$x \to$ all values $x > 0$

$y =$ all values $y \leq -1$

(b) Find the points where the curve has horizontal or vertical tangents, if any.

tangent line has slope $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = \dfrac{-2t}{-e^{-t}} = \dfrac{2t}{e^{t}}$

vert: none $(e^t > 0)$

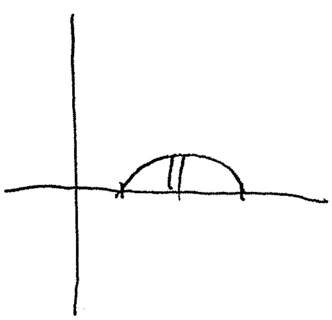
horiz: $t = 0$

Point: $(e^{-0}, 1 - 0^2) = (1, 1)$

(c) Find the area bound by this curve and the x axis (You'll get most of the credit for setting this one up).

x-axis $y = 0$ @ $t = 1, -1$

Note: curve runs RIGHT $\to$ LEFT



So Area $= \int y \, dx$

$= \int_{1}^{-1} (1 - t^2)(-e^{-t}) \, dt$

$= \int_{1}^{-1} t^2 e^{-t} - e^{-t} \, dt$

parts, parts

$= -t^2 e^{-t} - 2te^{-t} - 2e^{-t} + e^{-t} \Big|_{1}^{-1}$

$(-e + 2e - 2e + e) - \left(-\frac{1}{e} - \frac{2}{e} - \frac{2}{e} + \frac{1}{e}\right) = \frac{4}{e}$