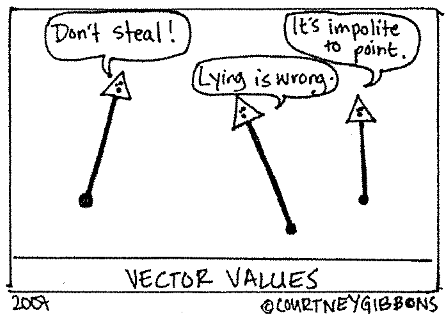
KEY

Math 225: Quiz the Second
February 6, 2015

This quiz is closed book and closed notes. Please justify all of your answers. You have until the end of the class period to finish.



1. Let **a**, **b**, and **c** be vectors. For each quantity, state whether it is a vector, a scalar, or nonsense.

   (a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$ → scalar value
   v · v

   v − s nonsence
   (b) $\mathbf{a} - (\mathbf{b} \cdot \mathbf{c})$

   (c) $\frac{\mathbf{a} \times \mathbf{b}}{\mathbf{a} \cdot \mathbf{b}}$ $\frac{v}{s}$ → vector (scaled)

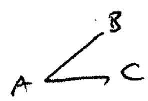
   (d) $\mathbf{a}^2$ → nonsense!
   v times v

2. Consider the points $A = (1, -1, 0)$, $B = (3, -3, 1)$ and $C = (-3, 1, 4)$

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{CB}$.

$\overrightarrow{AB} = \langle 2, -2, 1 \rangle$
$\overrightarrow{AC} = \langle -4, 2, 4 \rangle$
$\overrightarrow{CB} = \langle 6, -4, -3 \rangle$

(b) Is angle $\angle BAC$ acute, right, or obtuse? Explain.



$\rightarrow \quad \overrightarrow{AB} \cdot \overrightarrow{AC} = -8 - 4 + 4 = -8 \quad \leftarrow \quad \overrightarrow{AB} \cdot \overrightarrow{AC} < 0$

so OBTUSE.

(c) Find a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 2, -2, 1 \rangle \times \langle -4, 2, 4 \rangle = \langle -10, -12, -4 \rangle$$

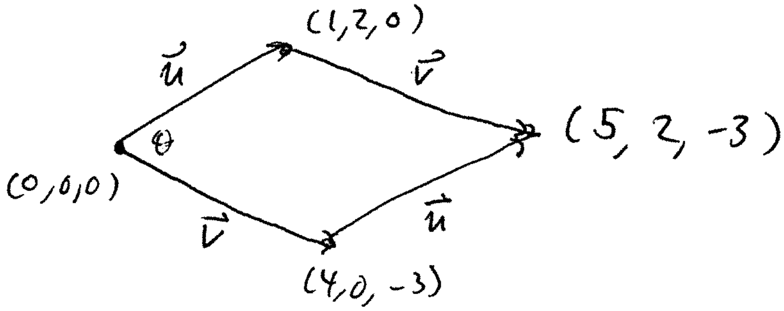
(d) Find a unit vector parallel to $\overrightarrow{CB}$.

$\overrightarrow{CB} = \langle 6, -4, -3 \rangle$

$$\text{Unit} = \frac{\overrightarrow{CB}}{|\overrightarrow{CB}|} = \frac{1}{\sqrt{61}} \langle 6, -4, -3 \rangle.$$

3. Three vertices of a parallelogram are at $(0,0,0)$, $(1,2,0)$, and $(4,0,-3)$

(a) Find the fourth vertex.

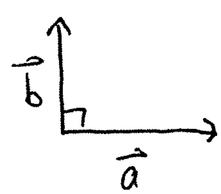


(b) Find the area of the parallelogram.

$$\text{Area} = |\vec{u} \times \vec{v}| = \left| \langle 1,2,0 \rangle \times \langle 4,0,-3 \rangle \right| = |\langle -6, 3, -8 \rangle| = \sqrt{109} \text{ units}^2$$

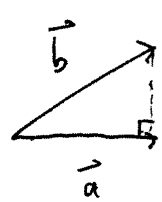
4. Draw examples of *distinct, nonzero* vectors **a** and **b** such that

(a) $\mathbf{proj_a b} = \mathbf{0}$



$\vec{b} \perp \vec{a}$.

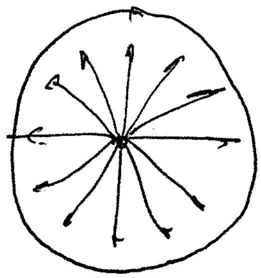
(b) $\mathbf{proj_a b} = \mathbf{a}$



$(\vec{b} - \vec{a}) \perp (\vec{a})$.

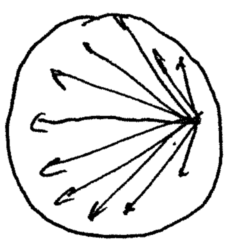
Make clear any geometric relationships between the vectors in your examples.

5. (a) The face of a clock has 12 vectors, each with their tail at the center, and each with their head on a different number. What is the sum of these vectors.



$\sum v_i = \vec{0}$

(b) The face of the clock has 11 vectors, each with their tail on '3', and each pointing to a different number. What is the sum of these vectors?



$\sum v_i$ points in the $-\vec{j}$ direction and is 6 times as long as the $3 \to 9$ vector

$\sum v_i = 6\,(\overrightarrow{3 \to 9})$.

PICK UP A 'BONUS' PROBLEM SHEET ON THE WAY OUT OF CLASS.