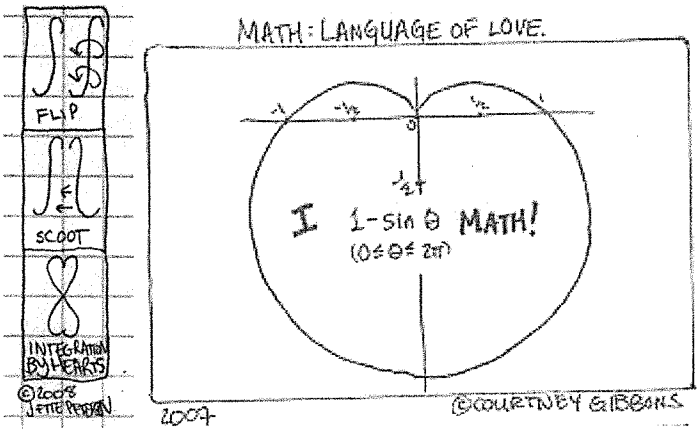
KEY

Math 225: Quiz the Third
February 13, 2015

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.

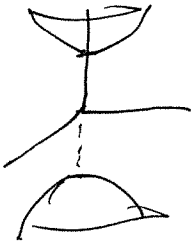


1. Identify the following surfaces, *and* answer the related questions (Hint: Put these in 'standard' form).

   (a) $32z^2 = 2x^2 + 72y^2 + 288$ (Give permissible values for z)

   $\frac{z^2}{9} = \frac{x^2}{144} + \frac{y^2}{4} + 1$ $\quad z \geq 3$ or $z \leq -3$

   Hyperboloid of 2 sheets

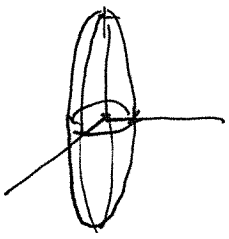


   (b) $32x^2 + 72y^2 + 2z^2 = 288$ (Give maximum values for x, y, and z).

   $\frac{x^2}{9} + \frac{y^2}{4} + \frac{z^2}{144} = 1$

   $-3 \leq x \leq 3$
   $-2 \leq y \leq 2$
   $-12 \leq z \leq 2$

   Ellipsoid

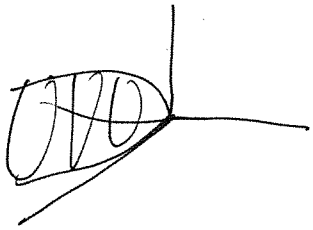


   (c) $18x^2 + 2y + 32z^2 = 0$ (Give the axis of symmetry and 'direction' of this graph)

   $2y = -18x^2 - 32z^2$

   $y = -9x^2 - 16z^2$

   elliptical Paraboloid on y-axis opening left



2. (a) Find the equation of the line containing the points (0,1,4) and (2,2,3). Give at least two different forms.

point → $(0,1,4)$

vector → "$\langle 2,2,3\rangle - (0,1,4)$" $= \langle 2,1,-1\rangle$

line $\vec{l}(t) = \langle 0,1,4\rangle + t\langle 2,1,-1\rangle$

or $x = 2t$, $y = 1+t$, $z = 4-t$ or $\frac{x}{2} = y-1 = \frac{z-4}{-1}$

(b) Find the equation of the plane perpendicular to the line in (a), through the point (-1,2,11).

$\vec{n} = \langle 2,1,-1\rangle$

point $(-1,2,11)$

plane $2(x+1) + (y-2) - (z-11) = 0$

$2x + y - z = -11$

(c) Find the point of intersection of the line in (a) and the plane in (b).

$x = 2t$, $y = 1+t$, $z = 4-t$

$2(2t) + (1+t) - (4-t) = -11$

$6t - 3 = -11$

$6t = -8$

$t = -4/3$

point $= \left(\frac{8}{3}, \frac{-1}{3}, \frac{16}{3}\right)$.

3. Consider the lines

$$\frac{x-1}{3} = \frac{y-2}{-6} = \frac{z+3}{3}$$

and

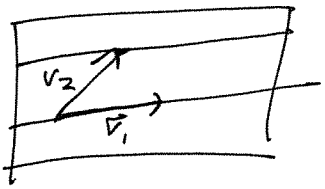
$$\frac{x-1}{-2} = \frac{y+1}{4} = \frac{z+2}{-2}$$

(a) Explain why the lines are parallel.

Because $\langle 3, -6, 3 \rangle = -\frac{3}{2}\langle -2, 4, -2 \rangle$

Their direction vectors are parallel

(b) Find the plane containing these two lines.



$\vec{v}_1 = \langle 3, -6, 3 \rangle$

$\vec{v}_2 =$ "$(1,2,-3) - (1,-1,-2)$" $= \langle 0, 3, -1 \rangle$

$$\vec{n} = \langle 3, -6, 3 \rangle \times \langle 0, 3, -1 \rangle$$

$$= \langle -3, 3, 9 \rangle$$

plane : $-3(x-1) + 3(y-2) + 9(z+3) = 0$.

4. (a) Find the point of intersection of the lines

$$x = t - 1,\ y = 10 - 3t,\ z = t$$

$$x = 4 + 3t,\ y = 5 + t,\ z = 1 - t$$

line 1 (t) = line 2 (s)

$x = t - 1 = 4 + 3s$

$y = 10 - 3t = 5 + s$

$z = t = 1 - s$

from z: $t = 1 - s$

x: $t - 1 = (1 - s) - 1 = 4 + 3s$

$-s = 4 + 3s \quad -4s = 4 \quad s = -1$

$t = 1 - (-1) = 2$

y: $10 - 3(2) = 5 + (-1) = 4$ ✓

Point $(1, 4, 2)$

$x = 1 \quad y = 4 \quad z = 2$

(b) Find the angle of intersection of these two lines. (More than one answer possible).

Angle $\theta = \arccos\left(\frac{\vec{v}_1 \cdot \vec{v}_2}{|\vec{v}_1||\vec{v}_2|}\right) = \arccos\left(\frac{\langle 1, -3, 1\rangle \cdot \langle 3, 1, -1\rangle}{\sqrt{11} \cdot \sqrt{11}}\right)$

$= \arccos\left(\frac{-1}{11}\right)$.

5. (Bonus) You may have half a point or one and a half points Extra Credit. Note: If more than 25% of you pick one and a half points, no one gets anything.