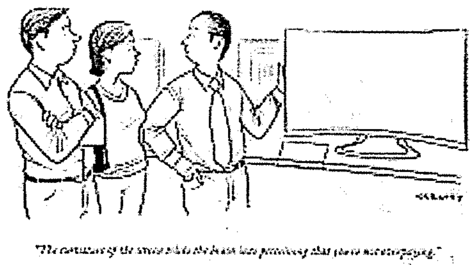
KEY

Math 225: Quiz the Fourth

February 27, 2015

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.



"The curvature of the screen tricks the brain into perceiving that you're not overpaying"

1. Let $\mathrm{a}(t) = \langle 2t, e^{2t}+5, -\sin(t)\rangle$. Also, let $\mathrm{v}(0) = \langle 0,0,4\rangle$ and $\mathrm{r}(0) = \langle 1,-1,0\rangle$. Find a formula for $\mathrm{r}(t)$.

$\vec{a}(t) = \langle 2t, e^{2t}+5, -\sin t\rangle$

$\vec{v}(t) = \langle t^2, \frac{e^{2t}}{2}+5t, \cos t\rangle + \langle x_0, y_0, z_0\rangle$

$\vec{v}(0) = \langle 0,0,4\rangle = \langle 0^2, \frac{1}{2}, 1\rangle + \langle x_0, y_0, z_0\rangle$

$x_0 = 0$

$y_0 = -\frac{1}{2}$

$z_0 = 3$

$\vec{v}(t) = \langle t^2, \frac{e^{2t}}{2} + 5t - \frac{1}{2}, \cos t + 3\rangle$

$\vec{r}(t) = \langle \frac{t^3}{3}, \frac{e^{2t}}{4} + \frac{5t^2}{2} - \frac{1}{2}t, \sin t + 3t\rangle + \langle x_1, y_1, z_1\rangle$

$\vec{r}(0) = \langle 1, -1, 0\rangle = \langle x_1, \frac{1}{4}+y_1, z_1\rangle$

$\vec{r}(t) = \left\langle \frac{t^3}{3}+1, \frac{e^{2t}}{4} + \frac{5t^2}{2} - \frac{1}{2}t - \frac{5}{4}, \sin t + 3t\right\rangle$

2. Suppose that an object in motion has zero curvature. Prove that the object is moving in a straight line.

$K(t) = 0 \Rightarrow \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} \Rightarrow |\vec{T}'(t)| = 0 \Rightarrow \vec{T}(t)$ is constant

$\vec{T}(t) = \langle a, b, c\rangle$

$\vec{r}(t) = \langle at + x_0, bt + y_0, ct + z_0\rangle$

linear!

3. Let $\mathbf{r}(t) = \langle t^2, \frac{2}{3}t^3, t\rangle$.

(a) Find $\mathbf{T}(1)$.

$\vec{r}\,'(t) = \langle 2t, 2t^2, 1\rangle \to \vec{r}\,'(1) = \langle 2, 2, 1\rangle \qquad |\vec{r}\,'(1)| = 3$

$\vec{T}(t) = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$

(b) If $\mathbf{N}(1)$ is parallel to $\langle -1, 2, -2\rangle$ find $\mathbf{N}(1)$ and $\mathbf{B}(1)$.

$\vec{N}(1) = \lambda\langle -1, 2, -2\rangle$ and is unit so

$\vec{N}(1) = \left\langle -\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right\rangle \qquad \vec{B} = \vec{T}\times\vec{N} = \begin{array}{r} \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle \\ \times\left\langle -\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}\right\rangle \\ \hline \left\langle \frac{-6}{9}, \frac{3}{9}, \frac{6}{9}\right\rangle = \left\langle \frac{-2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle \end{array}$

(c) Find $\kappa(t)$, the curvature as a function of time t. Note that your formula can be cleaned up so as not to involve any radicals. Please do this!

$$\kappa(t) = \frac{|r' \times r''|}{|r'|^3} \qquad \begin{array}{r} r' \ \langle 2t, 2t^2, 1\rangle \\ r'' \times \langle 2, 4t, 0\rangle \\ \hline \langle -4t, 2, 4t^2\rangle \end{array}$$

$$|r'| = \sqrt{4t^2 + 4t^4 + 1}$$

$$K(t) = \frac{\sqrt{16t^4 + 16t^2 + 4}}{\left(\sqrt{4t^4 + 4t^2 + 1}\right)^3} = \frac{2\sqrt{(2t^2+1)^2}}{\sqrt{(2t^2+1)^2}^{\,3}} = \frac{2}{(2t^2+1)^2}$$

4. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.) Nov. 19

(b) Of the 46 of us (myself included, and sisters counting only once!), what is the approximate probability that at least 2 of us have the same birthday?

i. 9 % ii. 18 % iii. 54 % iv. 95 %