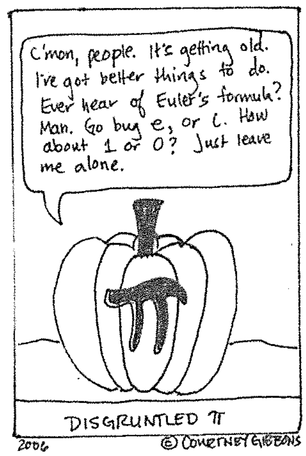
KEY

Math 126: Quiz the Fifth

$\pi - 1$

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Suppose that a colleague tells you that he has computed the following partial derivatives for a function $f(x, y)$

$$f_x(x, y) = 3x + 2y \qquad f_{xy} = 2$$
$$f_y(x, y) = 4x + y^2 \qquad f_{yx} = 4$$

$\neq$

   (a) Do you believe him? Why or why not?

   No. For well behaved functions $f_{xy} = f_{yx}$; it is not so here.

   (b) If he was right about f_y, give at least three functions that could be $f(x, y)$. Your functions should differ by more than just a constant.

$f_y = 4x + y^2$

$f = 4xy + \frac{y^3}{3} + g(x)$

so $4xy + \frac{y^3}{3} + x$

$4xy + \frac{y^3}{3} + 19$

$4xy + \frac{y^3}{3} + x\ln x + \cos x^3$

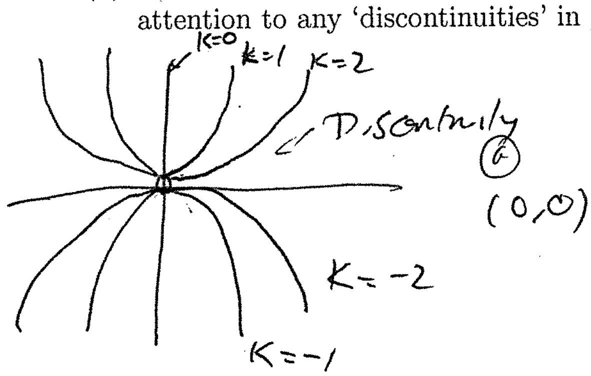
all ok

2. Let $f(x,y) = \dfrac{x^2}{y}$

(a) What is the domain of $f(x,y)$?

All (x,y) s.t. $y \neq 0$.

(b) Draw level curves for $f(x,y)$ corresponding to $k = -2, -1, 0, 1, 2$. Pay attention to any 'discontinuities' in your graphs.



$\dfrac{x^2}{y} = K \Rightarrow y = \dfrac{x^2}{K}$

Parabolas @ the origin

(c) Calculate the partials, $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

$f_x = \dfrac{2x}{y}$ $\qquad f_y = \dfrac{-x^2}{y^2}$

(d) Find the tangent plane at the point $(2,1)$ and use it to approximate $f(2.1, 0.95)$.

T plane @ (2,1)

$f(2,1) = \frac{4}{1} = 4$

$f_x(2,1) = \dfrac{2x}{y}\Big|_{(2,1)} = 4$

$f_y(2,1) = \dfrac{-x^2}{y^2}\Big|_{(2,1)} = -4$

$z = 4 + 4(x-2) - 4(y-1)$

$f(2.1, 0.95) \approx \cancel{4 + 4(2.1)}$

$4 + 4(2.1-2) - 4(0.95-1)$

$= 4 + .4 + .2$

$= 4.6$

3. Suppose that $z = x^3 + 3xy + y^3$, and that $x = u^2 + v^2$ and that $y = 2uv$. Find $\frac{\partial z}{\partial v}$.

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

$$= (3x^2+3y)(2v) + (3x+3y^2)(2u)$$

how it feels

4. The Wind Chill (perceived outside temperature on a windy day) W is a function of T, the real temperature, and V, the speed of the wind. What do you expect the signs of $\frac{\partial W}{\partial T}$ and $\frac{\partial W}{\partial V}$ to be, and why?

If T increases, it feels warmer out

so $\frac{\partial W}{\partial T} > 0$

If W increase, it feels colder out

so $\frac{\partial W}{\partial V} < 0$

5. Suppose that a sand crab is on a beach, where the temperature in degrees Centigrade is given by $T(x, y) = 70 - 3x^2 - 2y^2$.

   (a) How hot is the crab at the point (3,1)?

$$T(3,1) = 70 - 3(3)^2 - 2(1^2)$$
$$= 70 - 27 - 2 = 41°C$$

   (b) OUCH! What temperature should he move to *cool off* the fastest?

↳ direction of max decrease $= -\vec{\nabla} T$.

$$\vec{\nabla} T = \langle T_x, T_y \rangle$$
$$= \langle -6x, -4y \rangle \Big|_{(3,1)} = \langle -18, -4 \rangle$$

So $\langle 18, 4 \rangle$

6. (Bonus)

   (a) Favorite kind of pie? KEY LIME.

   (b) Favorite kind of pi?

$$\sum \frac{1}{K^2} = \frac{\pi^2}{6}.$$