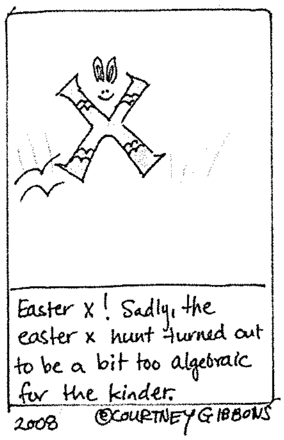
KEY

Math 126: Quiz the Sixth
April 3, 2015

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find and classify the critical points of $f(x,y) = 3x - x^3 - 12y + y^3$. (There are four).

$f_x = 3 - 3x^2 = 0 \qquad x = \pm 1$

$f_y = 3y^2 - 12 = 0 \qquad y = \pm 2$

cp's $(1,2)$ $(-1,2)$ $(1,-2)$ $(-1,-2)$

$D = (-6x)(6y) - 0^2 = -36xy$

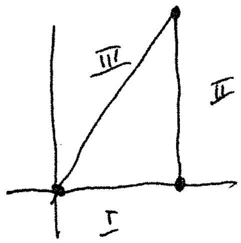
$D_{(1,2)} < 0 \Rightarrow (1,2)$ is a saddle point

$D_{(-1,2)} > 0 \Rightarrow (-1,2)$ is a max or min. $\quad f_{yy} = 6y > 0 \Rightarrow (-1,2)$ is a local min.

$D_{(1,-2)} > 0 \Rightarrow (1,-2)$ is a max or min $\quad f_{yy} = 6y < 0 \Rightarrow (1,-2)$ is a local max.

$D_{(-1,-2)} < 0 \Rightarrow (-1,-2)$ is a saddle point

2. Find the maximum and minimum values of the function $f(x, y) = x + y - xy$ on the triangular region bound by the points $(0, 0)$, $(2, 0)$ and $(2, 4)$.



Cp's $f_x = 1 - y = 0$ $\quad$ cp @ $(1,1)$ $\quad$ $f(1,1) = 1$ *

$f_y = 1 - x = 0$

G MAX = 2

Boundaries I: $y = 0 \to f(x,0) = x$ $\quad$ no cp's

Boundaries (endpts) $\quad f(0,0) = 0$ *

$f(2,0) = 2$ *

G min = -2

II $\quad x = 2 \quad f(2,y) = 2 + y - 2y = 2 - y$ $\quad$ no cp's

Boundaries (endpts) $\quad f(2,0) = 2$ *

$f(2,4) = -2$ *

III $\quad y = 2x \quad f(x, 2x) = x + 2x - 2x^2 = 3x - 2x^2$

$f' = 3 - 4x = 0$

cp @ $\frac{3}{4}$ $\quad f\left(\frac{3}{4}, \frac{3}{2}\right) = \frac{3}{4} + \frac{3}{2} - \frac{9}{8}$

$= \frac{9}{4} - \frac{9}{8} = \frac{9}{8}$ *

3. Use Lagrange Multipliers to find the maximum and minimum value of $f(x, y) = xy$ subject to the constraint $4x^2 + y^2 = 8$.

$f(x,y) = xy \qquad g(x,y) = 4x^2 + y^2 = 8$

$\vec{\nabla} f = \lambda \vec{\nabla} g$

$y = 8\lambda x \qquad y = 16\lambda^2 y \Rightarrow \lambda = \pm\frac{1}{4}$ (or $y = 0$)

$x = 2\lambda y \qquad \Downarrow$

$4x^2 + y^2 = 8 \qquad y = \pm 2x$

$4x^2 + 4x^2 = 8$

$\Rightarrow x = \pm 1 \longrightarrow xy = 2 \leftarrow$ max

$y = \pm 2 \qquad$ or

$xy = -2 \leftarrow$ min

5. A rectangular box with no top is to have a volume of 32 in^3. Find the dimensions of the box that minimize the surface area.

$V = lwh = 32 in^3 \qquad SA = lw + 2lh + 2wh$

Subst $\quad h = \frac{32}{lw}$

$SA = lw + 2l\frac{(32)}{lw} + 2w\frac{(32)}{lw}$

$= lw + \frac{64}{w} + \frac{64}{l}$

$SA_l = w - \frac{64}{l^2} = 0 \quad w = \frac{64}{l^2}$

$SA_w = l - \frac{64}{w^2} = 0 \quad l - \frac{1}{64}l^4 = 0$

$l = 0$ or $l^3 = 64 \to l = 4 \quad w = 4 \quad h = 2$

or Lagrange

optimize $\quad lw + 2lh + 2wh \quad$ s.t. $\quad lwh = 32$

$f_w = \quad l + 2h = \lambda lh \qquad \lambda lwh = lw + 2wh = lw + 2lh = 2lh + 2lw$

$f_l = \quad w + 2h = \lambda wh \to \qquad \Rightarrow l = w = 2h$

$f_h = \quad 2(l + w) = \lambda lw \qquad \Rightarrow l = w = 4; h = 2$

$4 \times 4 \times 2$

6. (Bonus) Imagine tossing a fair coin five times. Write down your results as a list of heads and tails.

For the next two problems, you may use either 'substitution' or LaGrange Multipliers.

4. Find the point on the plane $x + y + z = 2$ closest to the point (2, 0, -3).

Minimize

$D^2 = (x-2)^2 + y^2 + (z+3)^2$ s.t. $x+y+z=2$

$x - 2 = -y - z$

Substitution

$D^2 = (-y-z)^2 + y^2 + (z+3)^2$

$D_y = -2(-y-z) + 2y = 0$ $4y + 2z = 0$

$D_z: -2(-y-z) + 2(z+3) = 0$ $2y + 4z + 6 = 0$

$-6z = 12$

$z = -2$

$y = 1$

$x = 3$

$(3, 1, -2)$

or

Lagrange

$D^2 = (x-2)^2 + y^2 + (z+3)^2$ s.t. $x+y+z=2$

$2(x-2) = \lambda$

$2y = \lambda$

$2(z+3) = \lambda$

$x - 2 = y = z + 3$

$x + x - 2 + x - 5 = 2$

$3x - 7 = 2$

$3x = 9$

$x = 3 \rightarrow y = 1 \quad z = -2.$