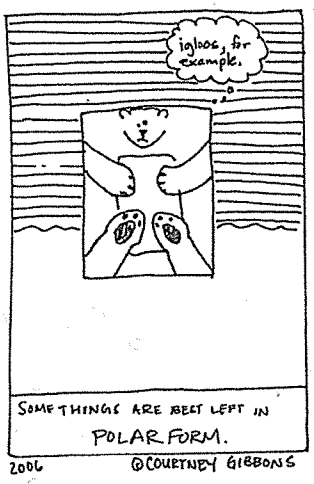
KEY

Math 126: Quiz the Seventh

April 3, 2015

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find

$$\iint_R x \sin(xy)\, dA$$

where $R = [0, \frac{\pi}{2}] \times [0, 2]$

$$\int_0^{\frac{\pi}{2}} \int_0^2 x \sin(xy)\, dy\, dx$$

$u = xy$

$du = x\, dy$

$$\int_0^{\pi/2} -\cos(xy)\Big|_0^{\pi/2} dx = \int_0^{\pi/2} -\cos 2x + 1 \; dx$$

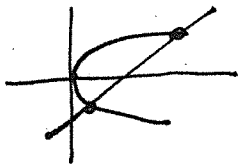
$$= -\frac{\sin 2x}{2} + x \Big|_0^{\pi/2} = \frac{\pi}{2}$$

2. Find

$$\iint_R y\,dA$$

where R is bound by $y = x - 2$ and $x = y^2$.

R



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$

$-1 \le y \le 2$

$y^2 \le x \le y+2$

$$\int_{-1}^{2}\int_{y^2}^{y+2} y\,dx\,dy$$

$$= \int_{-1}^{2} xy\Big|_{y^2}^{y+2} dy$$

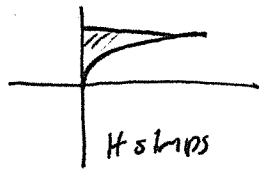
$$= \int_{-1}^{2} y^2 + 2y - y^3\,dy$$

$$= \frac{y^3}{3} + y^2 - \frac{y^4}{4}\Big|_{-1}^{2} = \frac{8}{3} + 4 - 4 - \left(\frac{-1}{3} + 1 - \frac{1}{4}\right)$$

$$\frac{8}{3} - \frac{5}{12} = \frac{27}{12}$$

3. Find

$$\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dy\,dx \rightarrow$$



$0 \le y \le 1$

$0 \le x \le y^2$

$$\int_0^1 \int_0^{y^2} \sqrt{y^3+1}\,dx\,dy$$

$$= \int_0^1 x\sqrt{y^3+1}\Big]_0^{y^2} dy$$

$$= \int_0^1 y^2\sqrt{y^3+1}\,dy = \frac{1}{3}\int_1^2 u^{1/2}\,du$$

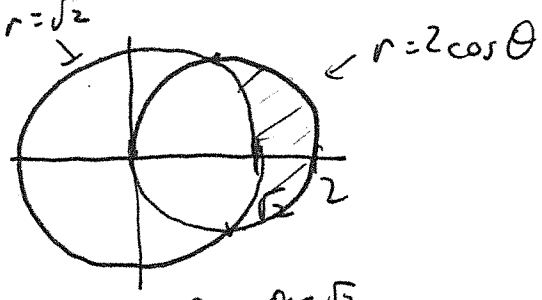
$u = y^3+1$ $\quad ll \Rightarrow 1$

$du = 3y^2\,dy$ $\quad ul \Rightarrow 2$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2}\Big|_1^2$$

$$= \frac{2}{9}\left(2^{3/2} - 1\right)$$

4. Find the area outside of the circle $r = \sqrt{2}$ and inside $r = 2\cos(\theta)$.



$2\cos\theta = \sqrt{2}$

$\cos\theta = \frac{\sqrt{2}}{2}$

$\theta = \pi/4$

$$\int_{-\pi/4}^{\pi/4} \int_{\sqrt{2}}^{2\cos\theta} 1 \cdot r \, dr \, d\theta$$

$$= \int_{-\pi/4}^{\pi/4} \frac{r^2}{2} \Big|_{\sqrt{2}}^{2\cos\theta} d\theta = \int_{-\frac{\pi}{4}}^{\pi/4} 2\cos^2\theta - 1 \, d\theta$$

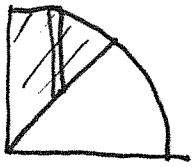
$$= \int_{-\pi/4}^{\pi/4} \cos 2\theta \, d\theta = \frac{\sin 2\theta}{2} \Big]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{2} - \left(-\frac{1}{2}\right) = 1$$

5. Find

$$\int_0^{\frac{3}{\sqrt{2}}} \int_x^{\sqrt{9-x^2}} x^2 + y^2 \, dy \, dx$$

by converting to polar coordinates.



$\rightarrow$ $0 \le r \le 3$

$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$$\rightarrow \int_{\frac{\pi}{4}}^{\pi/2} \int_0^3 r^2 \cdot r \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/2} \frac{r^4}{4} \Big|_0^3 d\theta = \int_{\pi/4}^{\pi/2} \frac{81}{4} d\theta$$

$$= \frac{81\pi}{16}$$

6. (Bonus) Tell me about a talk or poster that you went to as part of the undergraduate conference.