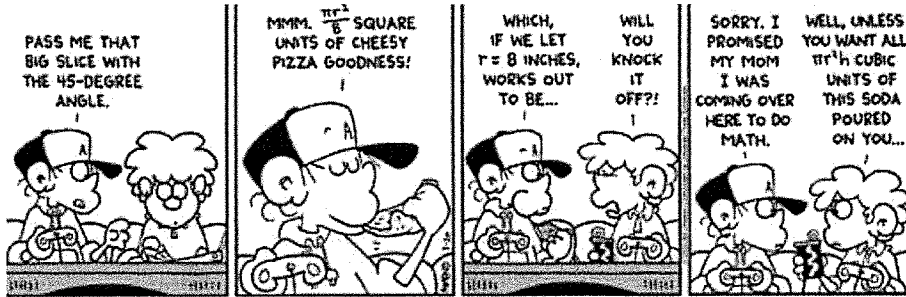


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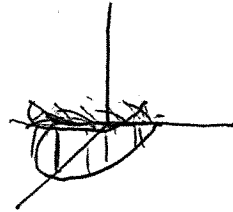
Math 126: Quiz the Eighth  
April 24, 2015

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



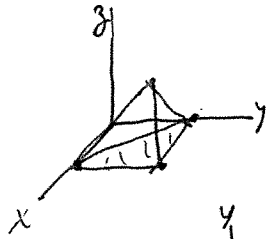
1. Set up the bounds for  $\iiint_E f(x, y, z) dV$  where  $E$  is bound by  $z = 0$ ,  $z = 2x$  and  $x = 4 - y^2$ . Do this *twice*, with different orders of integration each time.

$$\int_0^4 \int_{-\sqrt{4-x}}^{\sqrt{4-x}} \int_0^{2x} (f) dz dy dx$$



$$\int_{-2}^2 \int_0^{4-y^2} \int_0^{2x} (f) dz dx dy$$

2. Set up the bounds for  $\iiint_E f(x,y,z) dV$  where  $E$  is the tetrahedron with vertices  $(1,0,0)$ ,  $(0,1,0)$ ,  $(1,1,0)$  and  $(1,1,1)$ .

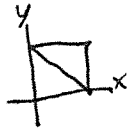


plane w/  $\langle 0,1,1 \rangle$   
and  $\langle 1,0,1 \rangle$

$$\vec{n} = \langle 1, 1, -1 \rangle$$

$$(x-1) + (y-1) - (z-1) = 0$$

$$x + y - z = 1$$

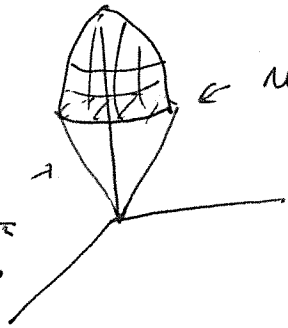


$$\int_0^1 \int_{1-x}^1 \int_0^{x+y-1} (f) dz dy dx$$

3. Find the volume bound below by the cone  $z = 2\sqrt{x^2+y^2}$  and above by  $z = 24 - x^2 - y^2$ .

P/boad  
 $24 - x^2 - y^2$   
 $= 24 - r^2$

Cone =  
 $2\sqrt{x^2+y^2}$   
 $= 2r$



Region = intersection

$$24 - r^2 = 2r$$

$$r^2 + 2r - 24 = 0$$

$$(r+6)(r-4) = 0$$

$$r = -6 \quad r = 4$$

$$\int_0^{2\pi} \int_0^4 \underbrace{(24 - r^2)}_{\text{p boad}} - \underbrace{2r}_{\text{cone}} r dr d\theta = \int_0^{2\pi} \int_0^4 (24r - r^3 - 2r^2) dr d\theta$$

$$= \int_0^{2\pi} \left[ 12r^2 - \frac{r^4}{4} - \frac{2r^3}{3} \right]_0^4 d\theta$$

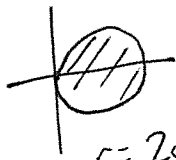
$$= \int_0^{2\pi} \left( 192 - 64 - \frac{128}{3} \right) d\theta$$

$$= \int_0^{2\pi} \left( \frac{384}{3} - \frac{128}{3} \right) d\theta = \frac{256 \cdot 2\pi}{3}$$

$$\frac{512\pi}{3}$$

4. (a) Express  $\iiint_E xyz \, dV$  where  $E$  is bound by  $z = 3$ ,  $z = 6$ , and the cylinder  $(x-1)^2 + y^2 = 1$  in cylindrical coordinates.

$$3 \leq z \leq 6$$



$$r = 2 \cos \theta$$

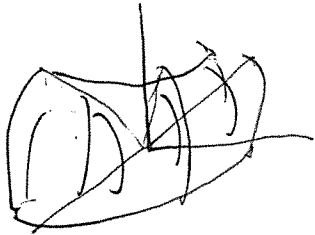
$$-\pi/2 \leq \theta \leq \pi/2$$

$$\int_{-\pi/2}^{\pi/2} \int_0^{2 \cos \theta} \int_3^6 (r^2 \sin \theta \cos \theta z) r \, dz \, dr \, d\theta$$

- (b) Is this integral positive, negative, or zero? Explain.

Integrand is  $y$ -balanced  $\Rightarrow \iiint xyz \, dV = 0$ .

5. Find the volume *inside* the top half of the sphere of radius 5, centered at the origin, but *below* the cone  $z = \sqrt{x^2 + y^2}$



spherical coords:

$$0 \leq \rho \leq 5$$

$$0 \leq \theta \leq 2\pi$$

$$\pi/4 \leq \phi \leq \pi/2$$

$$\begin{aligned} V &= \iiint_E 1 \, dV = \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \int_0^5 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \\ &= \int_0^{2\pi} \int_{\pi/4}^{\pi/2} \frac{125}{3} \sin \phi \, d\phi \, d\theta \\ &= \int_0^{2\pi} \frac{125}{3} \frac{\sqrt{2}}{2} \, d\theta = \frac{125\sqrt{2}}{3} \pi \end{aligned}$$

6. (Bonus) Pick a number at random from between 0 and 100. The person closest to  $\frac{2}{3}$  of the average will get the extra credit.