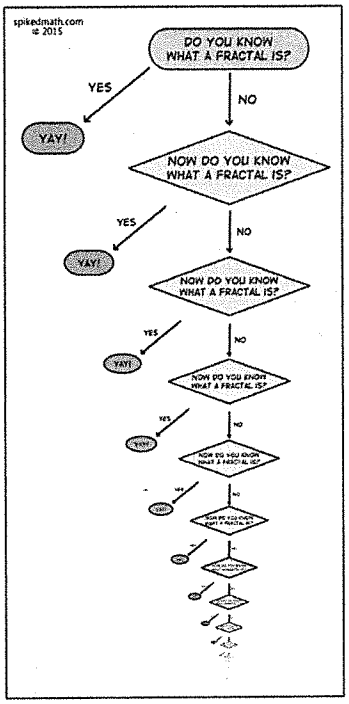
Math 225: Quiz the Penultimate

May 1, 2015

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find $\int_C \frac{2y}{x}\,ds$ where C is the parabolic segment of $y = x^2$ from $(1,1)$ to $(5,25)$.

$x = t$
$y = t^2$
$1 \le t \le 5$
$ds = \sqrt{1+4t^2}$

$$\int_1^5 \frac{2t^2}{t}\sqrt{1+4t^2}\,dt = \int_1^5 2t\sqrt{1+4t^2}\,dt$$

$$= \frac{1}{4}\left(\frac{2}{3}\left(1+4t^2\right)^{3/2}\right)\Bigg]_1^5$$

$$= \frac{1}{6}\left(101^{2/3} - 5^{2/3}\right)$$

2. (a) Find $\int_C f(x,y)\,ds$ where C is the circle of radius 2, and $f(x,y) = 2x + 3y + 6$.

$x = 2\cos t$
$y = 2\sin t$
$0 \le t \le 2\pi$

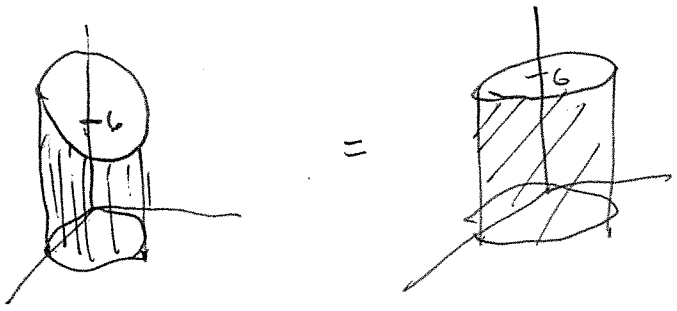
$ds = \sqrt{4\sin^2 t + 4\cos^2 t}$
$= 2$

$$\int_0^{2\pi} (4\cos t + 6\sin t + 6)\, 2\, dt$$
$$= \left[4\sin t - 6\cos t + 6 \right] \cdot 2 \Big]_0^{2\pi} = 24\pi$$

(b) Find $\int_C f(x,y)\,ds$ where C is the circle of radius 2, and $f(x,y) = 6$.

$$\int_0^{2\pi} 6(2)\, dt = 24\pi$$

(c) Comment, geometrically, on your answers for (a) and (b).



3. Find the amount of work done by $\mathbf{F}$ on an object moving along the line segment from $(2,0)$ to $(1,3)$ where $\mathbf{F} = \langle 2y, x + e^y \rangle$

$x = 2-t$
$y = 3t$
$0 \le t \le 1$

$W = \int \vec{F} \cdot d\vec{r} = \int_C 2y\,dx + (x+e^y)\,dy$

$\int_0^1 (6t)(-1) + (2-t+e^{3t})(3)\,dt$

$= \int_0^1 6 - 9t + 3e^{3t}\,dt = \left[6t - \frac{9}{2}t^2 + e^{3t}\right]_0^1$

$= 6 - \frac{9}{2} + e^3 - 1 = \frac{1}{2} + e^3$

4. For each field $\mathbf{F}$, find a function f such that $F = \nabla(f)$, or explain why none exists.

(a) $\mathbf{F} = \langle \cos x + 2xy, x^2 + \frac{1}{y} \rangle$

P Q $\quad Q_x = 2x = P_y$ ✓

$f(x,y) = \sin x + x^2 y + \ln|y| + C$

(b) $\mathbf{F} = \langle xe^{xy}, ye^{xy} \rangle$

$Q_x = y^2 e^{xy} \quad P_y = x^2 e^{xy}$

$\vec{F} \neq \vec{\nabla} f$ for any $\vec{F}$.

(c) $\mathbf{F} = \langle 2xy, x^2 + z^2, 2xz \rangle$

$P_y \overset{?}{=} Q_x \quad Q_z \overset{?}{=} R_y \quad R_x \overset{?}{=} P_z$

$2x = 2x \quad 2z \neq 0 \quad 2z \neq 0$

No such f exists.

$\int \vec{F} \cdot d\vec{r}$

5. Find $\int_C yz\ dx + xz\ dy + xy\ dz$ where C is the line segment from $(1, 2, 4)$ to $(3, 1, 6)$

Note $\vec{F} = \vec{\nabla} f$ where

$f = xyz$

So $\int_C \vec{\nabla} f \cdot d\vec{r} = xyz \Big|_{(1,2,4)}^{(3,1,6)} = 18 - 8 = 10$

P Q

6. Find $\oint_C (x \cos x + 2y)\ dx + (2x + \sqrt{y^3 + 1})\ dy$, where C is the triangle with vertices (0,0), (0,1), and (1,1).

$Q_x = 2 = P_y$ so $\vec{F}$ is conservative, C is closed $\Rightarrow$

Work $= 0$.

7. (Bonus)

   (a) Which topics do you feel most confident about heading into the final exam?

   (b) Which do you feel you need to work on?