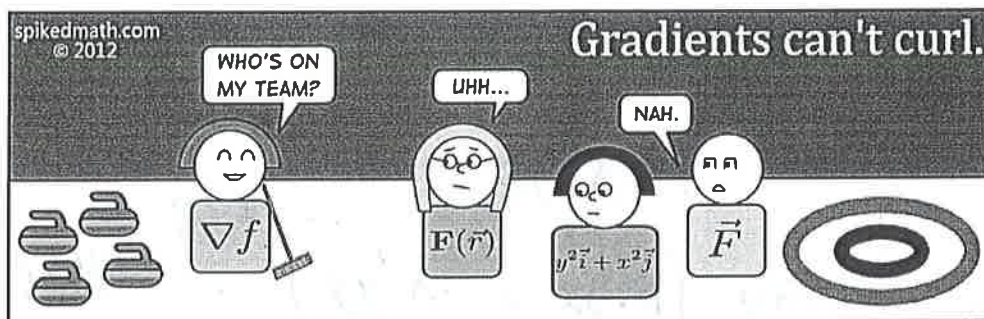


KEY

Math 225: Quiz the Last
May 5, 2017

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find $\int_C 2x \, dx + xy \, dy$ where C is the curve given by $\langle t^2, t^3 \rangle$ from $(0,0)$ to $(4,8)$. $0 \leq t \leq 2$

$$\int 2(t^2)(2t) + (t^2)(t^3)(3t^2) dt$$

$$= \int_0^2 4t^3 + 3t^7 dt = t^4 + \frac{3t^8}{8} \Big|_0^2$$

$$= 16 + \frac{3 \cdot 256}{8} = \underline{112}$$

2. Find $\int_C 2xyz \, dx + x^2z \, dy + x^2y \, dz$, where C is the line segment from $(1, 2, 3)$ to $(5, 1, 3)$.

$$\langle 2xyz, x^2z, x^2y \rangle = \vec{\nabla} f$$

where

$$f = x^2yz$$

$$\text{then } \int_C \vec{F} \cdot d\vec{r} = x^2yz \Big|_{(1,2,3)}^{(5,1,3)} = 75 - 6 = \underline{69}$$

3. Find a function f such that $\nabla(f) = \langle 2y + 3z, 2x + z, 3x + y \rangle$, or explain why none exists.

$$f_x = 2y + 3z$$

$$f = 2xy + 3xz + g(y, z)$$

$$f_y = 2x + g_y(y, z) = 2x + z$$

$$f_z = 3x + g_z(y, z) = 3x + y$$

$$g_y(y, z) = z$$

$$g_z(y, z) = y$$

$$g = yz$$

2

$$f = 2xy + 3xz + yz$$

4. Find $\oint_C (\cos(x) + \sin(y)) dx + (x \cos(y) + y^2) dy$, where C is the closed unit circle, centered at the origin, traversed counterclockwise from $(1,0)$.

P Q

$$Q_x = \cos y \quad P_y = \cos y$$

$\Rightarrow P, Q \rightarrow$ conservative

$$\rightarrow \oint_C \vec{F} \cdot d\vec{r} = 0.$$

5. Find $\oint_C (x+y^2) dx + (x^2+y) dy$ where C is the triangle from $(0,0)$ to $(2,0)$ to $(0,2)$ and back to $(0,0)$.

$Q_x = 2x \rightarrow$ f.s not conservative

$P_y = 2y$



Green's

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy$$

$$= \int_0^2 \int_0^{2-x} (2x - 2y) dy dx$$

$$= \int_0^2 (2xy - y^2) \Big|_0^{2-x} dx = \int_0^2 (2x(2-x) - (2-x)^2) dx$$

$$= \int_0^2 (4x - 2x^2 - 4 + 4x - x^2) dx$$

6. ('Bonus') What topics do you feel most confident about heading into the final?
What topics are you most concerned about?

$$= \int_0^2 (-3x^2 + 8x - 4) dx$$

$$= \left[-x^3 + 4x^2 - 4x \right]_0^2$$

$$= -8 + 16 - 8$$

$$= \underline{0}$$

