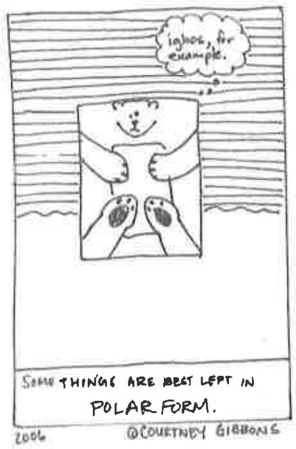
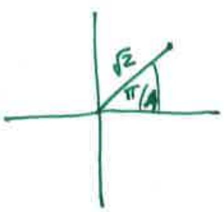
KEY

Math 225: Quiz the First
January 27, 2017

This quiz is closed book and closed notes. You may not use a calculator on this quiz. Please justify all of your answers. You have the remainder of the period.



1. (a) Determine rectangular coordinates for the polar point $(\sqrt{2}, \frac{\pi}{4})$.

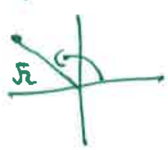


$x = \sqrt{2}\cos \pi/4 = 1$

$y = \sqrt{2}\sin \pi/4 = 1$

$(1, 1)$

(b) Determine rectangular coordinates for the polar point $(\sqrt{2}, \frac{3\pi}{4})$.

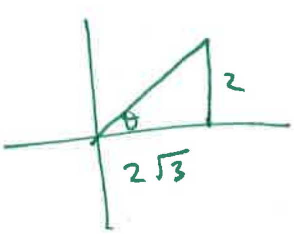


$x = \sqrt{2}\cos 3\pi/4$

$y = \sqrt{2}\sin 3\pi/4$

$= (-1, 1)$

(c) Determine polar coordinates for the rectangular point $(2\sqrt{3}, 2)$ (Please give THREE answers here, at least one of which has a negative r value).



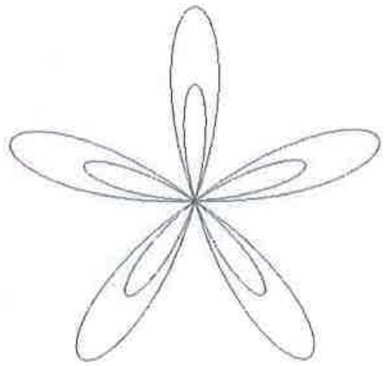
$r = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$

$\theta = \arctan\left(\frac{2}{2\sqrt{3}}\right) = \pi/6$

$(4, \pi/6), (4, 13\pi/6)$

$(-4, 7\pi/6)$

2. Below is the graph of $r = 1 + 4\sin(5\theta)$:



(a) What accounts for the number of leaves on this figure?

The multiplier 5 in $4 \sin 5\theta$

(b) How long are the outer leaves? How long are the inner leaves? How do you know?

$-1 \leq \sin 5\theta \leq 1$

$-4 \leq 4\sin 5\theta \leq 4$

$-3 \leq 1 + 4\sin 5\theta \leq 5$

outer leaves $\to 5$

inner leaves 3

3. Consider the function $r = 3 + 2\cos(\theta)$. Does this graph ever go through the origin? Why or why not?

$0 = 3 + 2\cos\theta \Rightarrow \cos\theta = \frac{-3}{2}$

This does not happen,

so NO.

4. Find the area inside one leaf of the rose $r = \sin(5\theta)$.

$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} (f(\theta))^2 \, d\theta$$

$$A = \frac{1}{2}\int_0^{\pi/5} \sin^2(5\theta)\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/5} \frac{1-\cos(10\theta)}{2}\, d\theta$$

$$= \frac{1}{2}\left(\frac{\theta}{2} - \frac{\sin 10\theta}{20}\right]_0^{\pi/5}$$

$$= \pi/20 - 0 = \pi/20$$

5. Find the slope of the tangent line of $r = \sin(5\theta)$ at $\theta = \frac{\pi}{10}$.

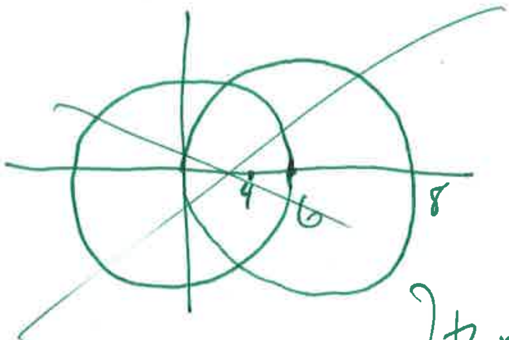
$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{+r\cos\theta + (\sin\theta)\frac{dr}{d\theta}}{-r\sin\theta + (\cos\theta)\frac{dr}{d\theta}} \to \frac{\cos\frac{\pi}{10} + 0}{-\sin\frac{\pi}{10} + 0}$$

$$= -\cot(\pi/10)$$

$$r = \sin \pi/2 = 1$$

$$\frac{dr}{d\theta} = 5\cos(5\theta)\Big|_{\pi/10} = 5\cos \pi/2 = 0$$

6. Draw a sketch of the graphs $r = 6$ and $r = 8\cos(\theta)$. (pay attention to the radii!)



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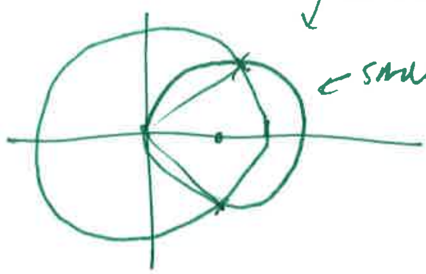

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7. Find the intersection points of these graphs.

$$\left(\frac{9}{2}, \frac{9[illegible]}{2}\right)$$

$$\left(\frac{9}{2}, -\frac{9[illegible]}{2}\right)$$

$$6 = 8\cos\theta \qquad \cos\theta = \frac{6}{8}$$

$$\theta = \arccos\left(\frac{3}{4}\right)$$

$$-\arccos\left(\frac{3}{4}\right)$$

8. Set up, but do not evaluate, the integral to find the area outside $r = 6$ but inside $r = 8\cos(\theta)$

$$\frac{1}{2}\int_{-\arccos\left(\frac{3}{4}\right)}^{\arccos\left(\frac{3}{4}\right)} (8\cos\theta)^2 - (6)^2 \, d\theta .$$