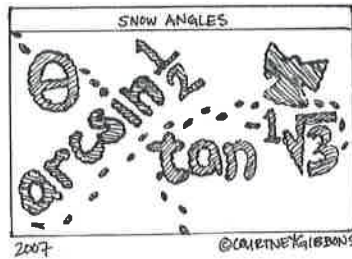


KEY

Math 225: Quiz the Second February 3, 2017

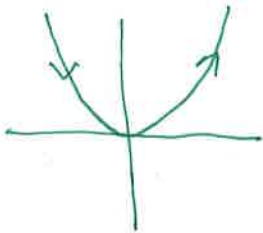
This quiz is closed book and closed notes. You may not use a calculator on this quiz. Please justify all of your answers. You have the remainder of the period.



1. Describe the similarities and differences between the parametric curves given by these equations. Here, t ranges over all real numbers. (Hint: think about permissible x and y values.)

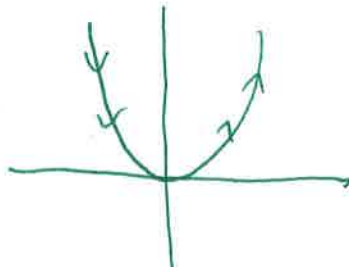
- (a) $x = t, y = t^2$
- (b) $x = 3t, y = 9t^2$
- (c) $x = \sin(t), y = \sin^2(t)$
- (d) $x = e^t, y = e^{2t}$

} all are parabolas of the form $y = x^2$



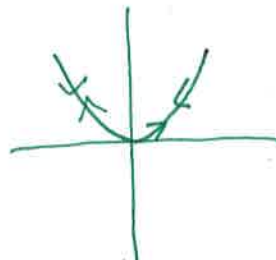
(a)

full parabola



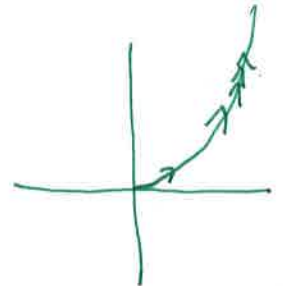
(b)

full parabola
"3x as fast"



(c)

restricted
parabola
 $-1 \leq x \leq 1$
 $0 \leq y \leq 1$
oscillating



(d)

half branch
 $x > 0$
 $y > 0$
increasing in
speed.

2. Consider the curve given by the parametric equations $x(t) = 2t, y(t) = 2t - t^2$.

(a) Find the *points*, if any, where the curve crosses the x -axis.

$$\begin{aligned} x\text{-axis} &\rightarrow y=0 & t=0 & t=2 \\ &\rightarrow 2t-t^2=0 & (0,0) & (4,0) \\ & t=0,2 \end{aligned}$$

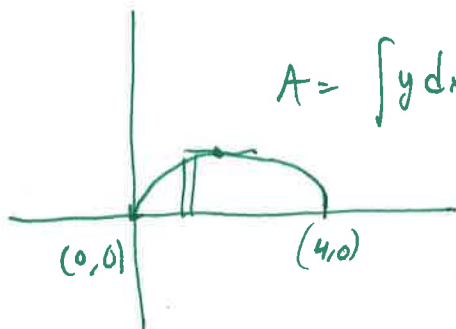
(b) Find the points, if any, where the curve has a horizontal tangent line.

$$\begin{aligned} \text{horiz tangent line} &\rightarrow \frac{dy}{dt}=0 \rightarrow 2-2t=0 \\ & t=1 \\ & (2, 1) \end{aligned}$$

(c) Find the points, if any, where the curve has a vertical tangent line.

$$\begin{aligned} \text{vert tangent line} &\rightarrow \frac{dx}{dt}=0 \rightarrow 2=0 \quad \ddot{} \\ & \text{none} \end{aligned}$$

(d) Find the area bound by this curve and the x -axis.

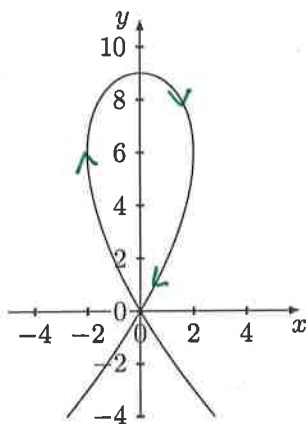


$$A = \int y dx = \int_0^2 (2t - t^2)(2) dt$$

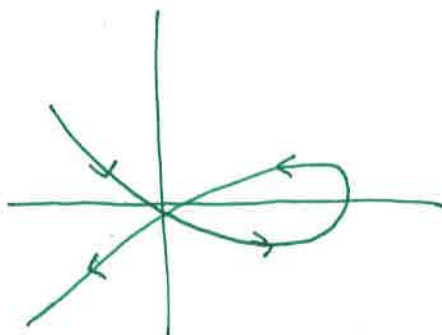
$$= 2 \left[t^2 - \frac{t^3}{3} \right]_0^2$$

$$= 2 \left[4 - \frac{8}{3} \right] = \underline{\underline{\frac{8}{3}}}$$

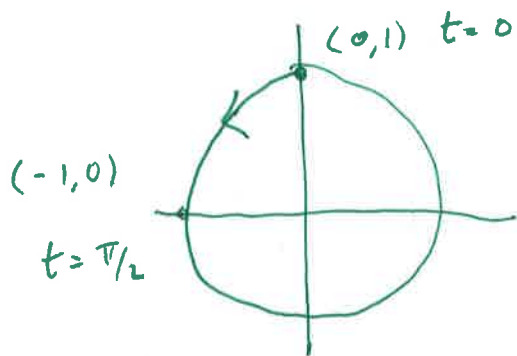
3. Recall the graph of the curve $x = 3t - t^3, y = 9 - 3t^2$.



Use this to draw the graph of the equations $x = 9 - 3t^2, y = 3t - t^3$. Please pay attention to the orientation (direction) of your curve!!



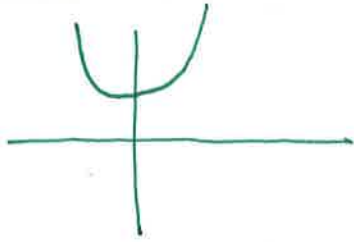
4. Give a parametrization of the unit circle, centered at the origin, which starts at the point $(0,1)$ and travels counterclockwise. Assume $0 \leq t \leq 2\pi$.



$$\langle -\sin t, \cos t \rangle \quad 0 \leq t \leq 2\pi$$

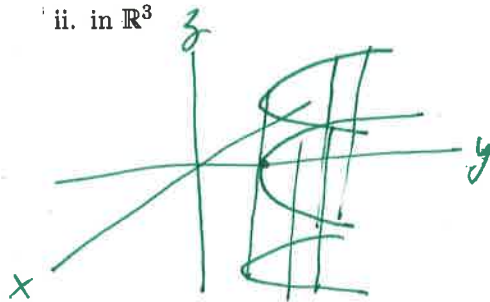
5. (a) Draw and describe the graph of the equation $y = x^2 + 1$ in

i. in \mathbb{R}^2



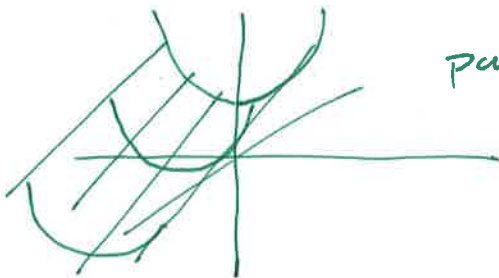
parabola, opening upward,
shifted up 1 unit from
the origin

ii. in \mathbb{R}^3



parabolic cylinder
parallel to z-axis.

(b) Draw and describe the graph of $z = y^2 + 1$ in \mathbb{R}^3 .



parabolic cylinder
parallel to x-axis.

Bonus:

1. What is the difference between a *trochoid* and a *cycloid*

2. The term *tautochrone* comes from the Greek words meaning what?