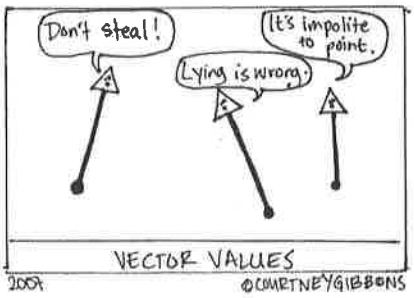
KEY

Math 225: Quiz the Third
February 10, 2017

This quiz is closed book and closed notes. Please justify all of your answers. You have until the end of the class period to finish.



1. Let $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ be vectors. For each quantity, state whether it is a vector, a scalar, or nonsense.

   (a) $(\mathbf{a}+\mathbf{b})\times\mathbf{c}$

   $\vec{v} \times \vec{v}$ = vector

   (b) $(\mathbf{a}\cdot\mathbf{b})+\mathbf{c}$

   $s + \vec{v}$ = nonsense

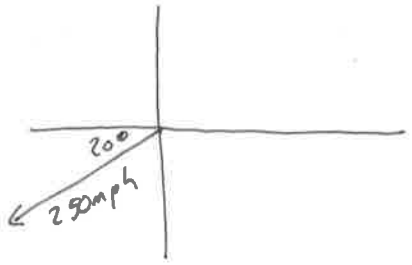
   (c) $\frac{\mathbf{a}}{|\mathbf{a}|}\cdot\mathbf{b}$

   $\vec{u} \cdot \vec{v}$ = scalar

   (d) $\mathbf{a}^2$

   nonsense → we don't square vectors

2. An airplane is traveling 20° south of west at 250 miles/hour. Find the horizontal and vertical components of its velocity.



$\text{horiz} = 250 \cos 200°$
$\text{or} -250 \cdot (\cos 20°)$

$\text{vert} = 250 \sin 200°$
$\text{or} \quad -250 \sin 20°$

3. Find all values x such that the vectors $\langle x, 3, 5\rangle$ and $\langle x, -2x, 1\rangle$ are orthogonal.

$\langle x, 3, 5\rangle \cdot \langle x, -2x, 1\rangle = x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5, 0$

4. Find a vector of length 4 parallel to $\langle 1, 2, 2\rangle$

$\langle 1,2,2\rangle \xrightarrow{\vec{u}} \langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3}\rangle \xrightarrow{\times 4} \langle \frac{4}{3}, \frac{8}{3}, \frac{8}{3}\rangle$

$\sqrt{1+4+4} = 9$

5. Consider the points $A = (1, 2, 3)$, $B = (2, 3, 2)$ and $C = (5, 1, 6)$

(a) Find the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$, and $\overrightarrow{CB}$.

$\overrightarrow{AB} = \langle 1, 1, -1 \rangle$

$\overrightarrow{AC} = \langle 4, -1, 3 \rangle$

$\overrightarrow{BC} = \langle 3, -2, 4 \rangle$

(b) Is angle $\angle BAC$ acute, right, or obtuse? Explain.

$\overrightarrow{AB} \cdot \overrightarrow{AC} = \langle 1, 1, -1 \rangle \cdot \langle 4, -1, 3 \rangle$

$= 4 - 1 - 3 = 0$

Right angle

(c) Find a vector perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 1, 1, -1 \rangle \times \langle 4, -1, 3 \rangle$

$\langle 3-1, -4-3, -1-4 \rangle = \langle 2, -7, -5 \rangle$

Check via Dot products

$2 - 7 + 5 = 0$

$8 + 7 - 15 = 0$

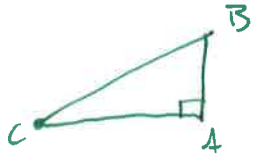
(d) Project $\overrightarrow{BC}$ onto $\overrightarrow{BA}$. What do you notice about your answer and why?

$$\text{Proj}_{\overrightarrow{BA}} \overrightarrow{BC} = \frac{\overrightarrow{BC} \cdot \overrightarrow{BA}}{|\overrightarrow{BA}|^2} \overrightarrow{BA}$$

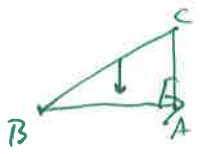
$$= \frac{\langle -1, -1, 1 \rangle \cdot \langle 3, -2, 4 \rangle}{|\langle -1, -1, 1 \rangle|^2} \langle -1, -1, 1 \rangle$$

$$= \frac{-3 + 2 + 4}{(\sqrt{3})^2} \langle -1, -1, 1 \rangle = \langle -1, -1, 1 \rangle$$

Since $\overrightarrow{BA} \perp \overrightarrow{BC}$ the projection of $\overrightarrow{BC}$ onto $\overrightarrow{BA}$ is just $\overrightarrow{BA}$



-1



6. Suppose that $\mathbf{v}$ and $\mathbf{w}$ are vectors, with $|\mathbf{v}| = 3$ and $|\mathbf{w}| = 7$.

(a) The maximum value of $\mathbf{v} \cdot \mathbf{w}$ is 21 and occurs when $\mathbf{v}$ and $\mathbf{w}$ are parallel.

$= |\vec{v}||\vec{w}| \cos\theta$

(b) The maximum value of $|\mathbf{v} \times \mathbf{w}|$ is 21 and occurs when $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

$|\vec{v}||\vec{w}| \sin\theta$

7. Let $\mathbf{a}$ and $\mathbf{b}$ be vectors. Prove that if $\mathbf{a} + \mathbf{b}$ is orthogonal to $\mathbf{a} - \mathbf{b}$, then $\mathbf{a}$ and $\mathbf{b}$ have the same length.

If $(\vec{a}+\vec{b}) \perp (\vec{a}-\vec{b})$

then $(\vec{a}+\vec{b}) \cdot (\vec{a}-\vec{b}) = 0$

then $(\vec{a}\cdot\vec{a}) + \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 0$

$\vec{a}\cdot\vec{a} - \vec{b}\cdot\vec{b} = 0$

$\rightarrow \vec{a}\cdot\vec{a} = \vec{b}\cdot\vec{b}$

$|\vec{a}|^2 = |\vec{b}|^2 \rightarrow |\vec{a}| = |\vec{b}|$

(Bonus) You may have half a point or one-and-a-half points extra credit on this quiz. Note: If more that $\frac{1}{4}$ of you choose one-and-a-half points, no one gets anything.