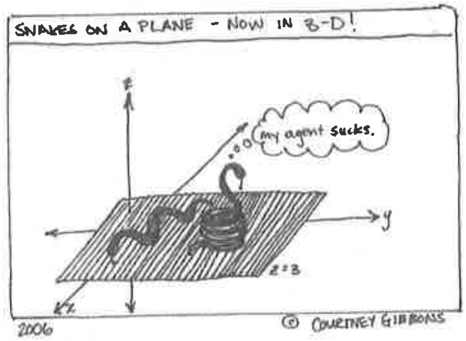
KEY

Math 225: Quiz the Fourth
February 17, 2017

This quiz is closed book and closed notes. Please justify all of your answers. You have the remainder of the period.



1. Identify the following surfaces, *and* answer the related questions.

   (a) $2x^2 + 32y^2 + 72z^2 = 288$ (Give maximum values for x, y, and z).

Ellipsoid

$$\frac{x^2}{144} + \frac{y^2}{9} + \frac{z^2}{4} = 1 \rightarrow$$

$x_{max} = 12$

$y_{max} = 3$

$z_{max} = 2$.

   (b) $2x^2 - 32y^2 - 72z^2 = -288$ (Give permissible values for x)

$2x^2 + 288 = 32y^2 + 72z^2$ hyp of <u>One</u> sheet

All real x are permissible

$x = \pm\sqrt{32y^2 + 72z^2 - 288}$.

2. Find the equation of the line containing the points (4,1,2) and (-2, 2, 4). Give at least two different forms.

point $(4,1,2)$

vector $\vec{v} = \langle -6, 1, 2 \rangle$

line : $\vec{\ell}(t) = \langle 4,1,2 \rangle + t\langle -6,1,2 \rangle$

parametric
$$x = 4-6t$$
$$y = 1+t$$
$$z = 2+2t$$

symmetric $\dfrac{x-4}{-6} = \dfrac{y-1}{1} = \dfrac{z-2}{2}$

3. (a) Find the equation of the plane through (1,-3,2) perpendicular to the line $\mathbf{r}(t) = \langle 2, 4, 1 \rangle + t\langle 5, 1, -2 \rangle$

Point : $(1, -3, 2)$

$\vec{n} = \langle 5, 1, -2 \rangle$

plane : $5(x-1) + 1(y+3) - 2(z-2) = 0.$

$$= 5x + y - 2z = -2$$

(b) Find the distance from this plane to the point (2,1,3)

$$D = \frac{|ax+by+cz-d|}{\sqrt{a^2+b^2+c^2}} = \frac{|5\cdot 2 + 1\cdot 1 + (-2)(3) - (-2)|}{\sqrt{5^2+1^2+(-2)^2}}$$

$$= \frac{7}{\sqrt{30}}.$$

4. Consider the lines

$$\mathbf{r}(t) = \langle 2, 4, 1\rangle + t\langle 2, -1, 3\rangle$$

and

$$x = 4 - 4t,\ y = 2t - 1\ z = -5 - 6t$$

(a) Explain why the lines are parallel.

$\vec{r}_1$ has vector $\langle 2, -1, 3\rangle$ ↙ these are scalars of eachother...

$\vec{r}_2$ has vector $\langle -4, 2, -6\rangle$

(b) Find the equation of the plane containing both lines. (Hint: You'll need to begin by finding two *nonparallel* vectors on this plane)

Plane contains $\langle 2, -1, 3\rangle$

and the vector between the two points

$(2,4,1)$ to $(4,-1,-5)$

is $\langle 2, -5, -6\rangle$

$$\begin{array}{r} \langle 2, -1, 3\rangle \\ \times \langle 2, -5, -6\rangle \\ \hline \langle 6+15, 6+12, -10+2\rangle \\ = \langle 21, 18, -8\rangle \end{array}$$

Plane: $21(x-2) + 18(y-4) - 8(z-1) = 0$.

5. (a) Find the point of intersection of the lines

$$\mathbf{r}_1(t) = \langle 3+t, 3+2t, 2+t\rangle$$

$$\mathbf{r}_2(s) = \langle 3-2s, -s, s-1\rangle$$

$3+t = 3-2s$

$3+2t = -s$

$2+t = s-1$

$\hookrightarrow t = s-3$

$3+2(s-3) = -s$

$3+2s-6 = -s$ $\quad 3s = 3 \quad s=1,\ t=-2$

$3-2(1) = 1$

$-1 = -1$

$1-1 = 0$

$(1,-1,0)$

(b) Let $A = (1,4,3)$ and $B = (5,-2,5)$. Find the set of points $P = (x,y,z)$ such that the distance AP is twice as much as the distance BP. Simplify your equations and identify your surface.

$d(A,P) = 2\,d(B,P)$

$\sqrt{(x-1)^2+(y-4)^2+(z-3)^2} = 2\sqrt{(x-5)^2+(y+2)^2+(z-5)^2}$

$x^2-2x+1+y^2-8y+16+z^2-6z+9 = 4(x^2-10x+25+y^2+4y+4+z^2-10z+25)$

$3x^2-38x+3y^2+24y+3z^2-34z = k$

This is a sphere!

6. Extra Credit:

(a) What is your birthday? (Month and Day only. No years. Please.)

(b) Of the 56 of us (myself included), what is the approximate probability that at least 2 of us have the same birthday?

i. 16 % ii. 32 % iii. 64 % iv. 99 %