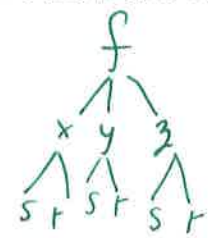
KEY

Math 225: Quiz the Fifth
Due Friday, 3/10/17

For this quiz, you may use your book and your notes, but you are expected to work independently on the questions. Please submit this quiz to my mailbox in the Olin Division Office by Friday at 5PM.

1. Suppose that $T = f(x, y, z)$ and that x, y and z are each functions of s and t. Give a formula for the partial derivative $\dfrac{\partial T}{\partial s}$.

$$\frac{\partial T}{\partial s} = \frac{\partial T}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial T}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial T}{\partial z}\frac{\partial z}{\partial s}$$



$F(x,y,z) = 9$

2. Let $x^3 + 2xyz + y^2 + z^2 = 9$. Calculate $\frac{\partial z}{\partial x}$ and $\frac{\partial y}{\partial x}$. State also the conditions under which your formulas are valid.

$$\frac{\partial z}{\partial x} = \frac{-F_x}{F_z} = \frac{-(3x^2+2yz)}{2xy+2z}$$ when $F_z \neq 0$

$$\frac{\partial y}{\partial x} = \frac{-F_x}{F_y} = \frac{-(3x^2+2yz)}{2xz+2y}$$ whn $F_y \neq 0$

3. Suppose that a function $f(x, y)$ has the partial derivative $f_x = x^2 + 2xy$. Give at least three different possibilities for f. (Your answers should differ by more than just a constant).

$f_x = x^2 + 2xy$

$f = \int x^2 + 2xy\,dx = \frac{x^3}{3} + x^2y + g(y)$

so $\frac{x^3}{3} + x^2y + y^3$ or $\frac{x^3}{3} + x^2y + e^y$ or $\frac{x^3}{3} + x^2y + \text{anything}$

4. Recall the ideal gas law,

$$PV = nRT,$$

where P represents pressure, T temperature, V volume, and n (number of moles of gas) and R (the 'ideal gas constant') are constants.

(a) If T is in Kelvin, P is in atmospheres, and V is in liters, what are the units on R?

$R = \frac{PV}{nT} = \frac{atm \cdot L}{mol \cdot K}$

(b) Suppose we have one mole of gas at 290K with a pressure of .5 atm. Find the approximate change in volume if we increase the temperature to 295K and decrease the pressure to .47 atm. (Assume $R = 0.082057$).

$V = \frac{nRT}{P}$

$dV = V_T\,dT + V_P\,dP$

$= \frac{nR}{P}\,dt + \left(-\frac{nRT}{P^2}\right)dP$

$= \frac{1 \cdot (.082057)}{.5}(+5) + \frac{-1(.082057)}{(.5)^2}(-.03) =$

$.082057\,(10 + .12) = .8304168 4\ L.$

5. Let $f(x, y) = 3x^2 - 2xy + y^3$.

(a) Find the equation of the tangent plane to f at the point $(-2, 1)$.

$f(-2,1) = 3(-2)^2 - 2(-2)(1) + (1)^3 = 17$

$f_x = 6x - 2y \Big|_{(-2,1)} = -14$

$f_y = -2x + 3y^2 \Big|_{(-2,1)} = 7$

$\underline{z = 17 - 14(x+2) + 7(y-1)}$

(b) Use your tangent plane to approximate $f(-2.02, 1.03)$

$f(-2.02, 1.03) \approx 17 - 14(-2.02+2) + 7(1.03-1)$

$= 17 + .28 + .21 = 17.49.$

(c) Find $\nabla(f)$ at the point $(-2, 1)$.

$\vec{\nabla} f \Big|_{(-2,1)} = \langle -14, 7 \rangle.$

$(-2,1)$ to $(0,0)$

(d) Find $D_{\mathbf{u}}f$ where $\mathbf{u}$ points from $(-2, 1)$ towards the origin.

$u = \langle 2, -1 \rangle \xrightarrow{\text{unit}} \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$

$D_{\vec{u}} f = \vec{\nabla} f \cdot u = \langle -14, 7 \rangle \cdot \left\langle \frac{2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle = \frac{-28}{\sqrt{5}} - \frac{7}{\sqrt{5}} = \frac{-35}{\sqrt{5}}$

(e) Find and classify the critical points of $f(x, y)$ (there are two).

$f_x = 6x - 2y = 0 \rightarrow x = \frac{1}{3}y$

$f_y = -2x + 3y^2 = 0$

$-\frac{2}{3}y + 3y^2 = 0$

$9y^2 - 2y = 0 \qquad y = 0, \frac{2}{9}$

$x = \frac{1}{3}y = 0, \frac{2}{27}$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= (6)(6y) - (-2)^2$

$= 36y - 4$

@ $(0,0)$ $\quad$ $D < 0 \Rightarrow$ Saddle point

@ $\left(\frac{2}{27}, \frac{2}{9}\right)$ $\quad$ $D = \frac{72}{9} - 4 > 0$

since $f_{xx} > 0$

Concave up

$\rightarrow$ local min

$F(x,y,z)$

6. Suppose that $x^2 + 2xyz + xz^2 - y^2z = 1$. Find the tangent plane to this surface at the point $(1, 1, -1)$.

$\vec{\nabla} F = \langle 2x + 2yz + z^2, 2xz + 2yz, 2xy + 2xz - y^2 \rangle \Big|_{(1,1,-1)}$

$= \langle 1, 0, -1 \rangle$

Plane: $1(x-1) + 0(y-1) - 1(z+1) = 0$

7. A beachgoer who can't stop thinking about Calc 3 realizes that the temperature function (in degrees Celsius) of the sand is given by $T(x, y) = 100 - 5x^2 - 10y^2$. She is standing at the point (1,2).

(a) How hot are her feet at this point?

$C(1,2) = 100 - 5(1)^2 - 10(2)^2 = 100 - 5 - 40$

$= 55\,°C$

(b) OUCH! In what direction should she run to cool her feet off the fastest?

To cool the fastest → travel as $-\vec{\nabla} T$

$= -\langle -10x, -20y \rangle \Big|_{(1,2)}$

$= -\langle -10, -40 \rangle$

$= \langle 10, 40 \rangle$