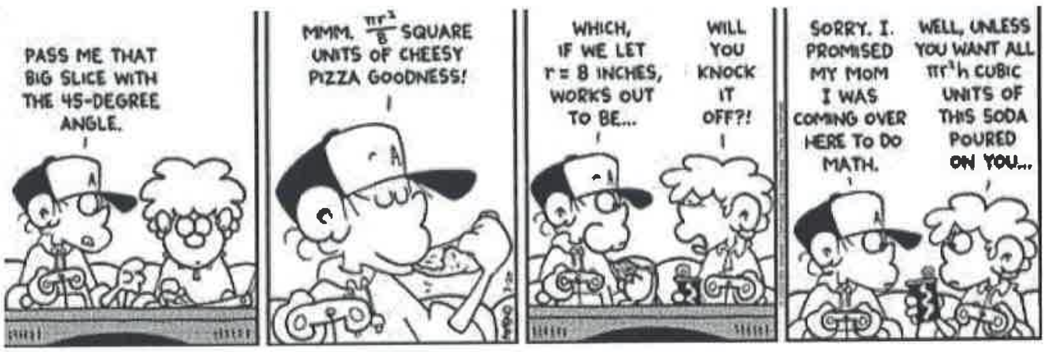
KEY

Math 225: Quiz the Sixth
March 31, 2017

You have the remainder of the period to complete this quiz. You may use a calculator for arithmetic and calculation only (i.e., no graphing!)



1. Find the maximum value of $\underset{f}{f(x,y,z) = x - y + z}$ on the sphere $\underset{g}{x^2 + y^2 + z^2 = 81}$.

Lagrange!

$$\vec{\nabla} f = \lambda \vec{\nabla} g$$

$$1 = 2\lambda x$$

$$-1 = 2\lambda y$$

$$1 = 2\lambda z$$

$$x = \frac{1}{2\lambda} \quad y = \frac{-1}{2\lambda} \quad z = \frac{1}{2\lambda}$$

$$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 81$$

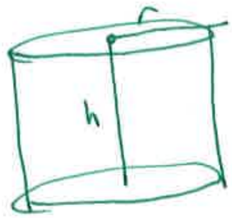
$$\frac{3}{4\lambda^2} = 81$$

$$\lambda^2 = \frac{3}{4 \cdot 81} \quad \lambda = \pm\frac{\sqrt{3}}{18}$$

For a max,

$$\left.\begin{aligned} x &= 9/\sqrt{3} \\ y &= -9/\sqrt{3} \\ z &= 9/\sqrt{3} \end{aligned}\right\} \quad \text{Value} = \frac{27}{\sqrt{3}} = 9\sqrt{3}.$$

2. (a) The new *UNCONSTRAINED* energy drink will come in a one liter can. What is the ratio of the height to the radius of this can if it uses the least material to make? (Comment also on why this ratio makes geometric sense).



$$\overset{g}{V = 1L = \pi r^2 h} \qquad \overset{f}{SA = 2\pi r^2 + 2\pi r h}$$

$$f_r = 4\pi r + 2\pi h = \lambda(2\pi r h)\, g_r\, \lambda$$

$2r = h$

→ Can is "squarish"

$$f_h = 2\pi r = \lambda(\pi r^2)\ g_h$$

$$\to \lambda = \frac{2}{r}$$

$$4\pi r = 2\pi h \to 2r = h$$

$$4\pi r + 2\pi h = \lambda(2\pi r h) = \frac{2}{r}(2\pi r h) = 4\pi h$$

(b) The drink comes in two flavors, Lambda Lightning (L) and Delta Dynamite (D). The cost for production of these cans is

$$C(L, D) = 2L^2 + LD + 3D^2 + 200.$$

If we want to produce 80 cans total, find the minimum cost.

$C = 2L^2 + LD + 3D^2$ $\qquad$ $L + D = 80$

$C_L = 4L + D = \lambda$

$C_D = L + 6D = \lambda$

$4L + D = L + 6D$

$3L = 5D$

$3L + 3D = 240$

$8D = 240$

$D = 30$

$L = 50$

$C = 2(50)^2 + 30 \cdot 50 + 3(30)^2 = 5000 + 1500 + 2700 = \$9200$

(c) Use your work in (b) to approximate the minimum cost of producing 82 cans.

$\lambda = 4L + D = 230$ ← marginal cost per can

Approx Cost → $\$9200 + 2(230) = \$9460$

3. Find $\iint_R 5x + 4y \, dA$ over the rectangle $R = [0,1] \times [3,4]$

$$\int_0^1 \int_3^4 5x + 4y \, dy \, dx$$

$$= \int_0^1 5xy + 2y^2 \Big]_3^4 dx$$

$$= \int_0^1 (20x + 32) - (15x + 18) \, dx$$

$$= \int_0^1 5x + 14 dx = \frac{5x^2}{2} + 14x \Big]_0^1 =$$

$$\frac{5}{2} + 14 = 33/2.$$

4. Find $\iint_R ye^{xy} \, dA$ over the rectangle $R = [0,2] \times [0,2]$.

order for u-sub

$$\int_0^2 \int_0^2 ye^{xy} \, dx \, dy \qquad u = xy \qquad du = y \, dx$$

$$\int_0^2 e^{xy} \Big|_0^2 dy = \int_0^2 e^{2y} - 1 \, dy$$

$$= \frac{e^{2y}}{2} - y \Big]_0^2 = \left(\frac{e^4}{2} - 2\right) - \left(\frac{1}{2} - 0\right)$$

$$= \frac{e^4}{2} - 5/2.$$

5. (Bonus) Choose a number between 0 and 100. One point extra credit goes to the student (or students) closest to 2/3 of the average. (You may also earn 1 point EC by justifying your choice of number).