KEY

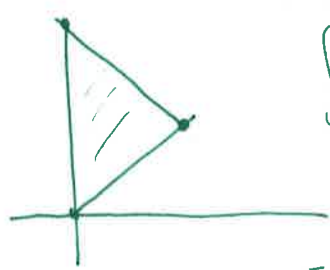
Math 126: Quiz the Seventh

April 7, 2017

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!

1. Draw the region of integration and evaluate

$$\int_0^1 \int_x^{2-x} x^2 - y \; dy \; dx$$



$$\int_0^1 x^2 y - \frac{y^2}{2}\Big|_x^{2-x} dx$$

$$= \int_0^1 x^2(2-x) - \frac{(2-x)^2}{2} - x^3 + \frac{x^2}{2}\, dx$$

$$= \int_0^1 2x^2 - x^3 - 2 + 2x - \frac{x^2}{2} - x^3 + \frac{x^2}{2}\, dx$$

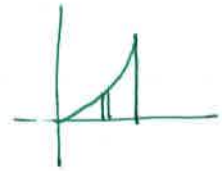
$$= \int_0^1 -2x^3 + 2x^2 + 2x - 2\, dx$$

$$= -\frac{2x^4}{4} + \frac{2x^3}{3} + x^2 - 2x \Big|_0^1$$

$$= -\frac{1}{2} + \frac{1}{3} + 1 - 2 = \frac{4}{3} - \frac{5}{2} = -\frac{7}{6}$$

2. Find

$$\int\int_R x\cos(y)\, dA$$



whre R is bound by $y=0$, $y=x^2$, and $x=1$.

$$\int_0^1\int_0^{x^2} x\cos y \, dy\, dx$$

$$= \int_0^1 x\sin y\Big]_0^{x^2} dx = \int_0^1 x\sin x^2\, dx = -\frac{1}{2}\cos x^2\Big]_0^1$$

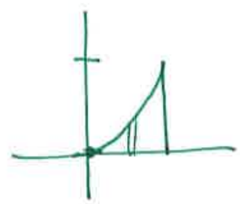
$$= \frac{-\cos(1)}{2} + \frac{1}{2}$$

$$= \frac{1-\cos(1)}{2}$$

3. Find

$$\int_0^1\int_{\sqrt{y}}^1 \sqrt{x^3+1}\, dx\, dy$$

flip the strip . . .



$$= \int_0^1\int_0^{x^2} \sqrt{x^3+1}\, dy\, dx$$

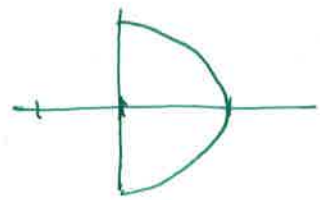
$$= \int_0^1 x^2\sqrt{x^3+1}\, dx = \left[\frac{1}{3}\cdot\frac{2}{3}(x^3+1)^{3/2}\right]_0^1$$

$$= \frac{2}{9}\left[2^{3/2}-1\right].$$

4. Convert

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} x \, dy \, dx$$

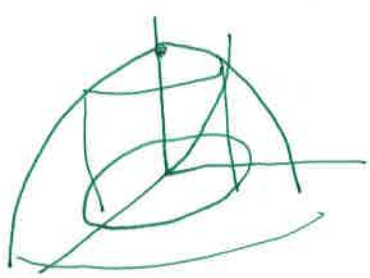
to polar coordinates and evaluate.



$$\int_{-\pi/2}^{\pi/2} \int_0^1 (r\cos\theta)\, r \, dr \, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1}{3}\cos\theta \, d\theta = \frac{1}{3}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} = 2/3$$

5. Find the volume inside both the cylinder $x^2 + y^2 = 4$ and $4x^2 + 4y^2 + z^2 = 64$.



$$z = \sqrt{64 - 4x^2 - 4y^2} = \sqrt{64 - 4r^2}$$

$$\int_0^{2\pi} \int_0^2 \sqrt{64-4r^2}\, r \, dr \, d\theta$$

$$= \int_0^{2\pi} -\frac{1}{8}\left(\frac{2}{3}(64-4r^2)^{3/2}\right)\Big|_0^2 d\theta \quad \text{switch}$$

$$= \int_0^{2\pi} \frac{1}{12}\left[64^{3/2} - 48^{3/2}\right] d\theta = \frac{\pi}{6}\left[512 - 48^{3/2}\right]$$

(Bonus) Pretend to toss a coin 5 times. Record your imagined tosses as a string of heads and tails.