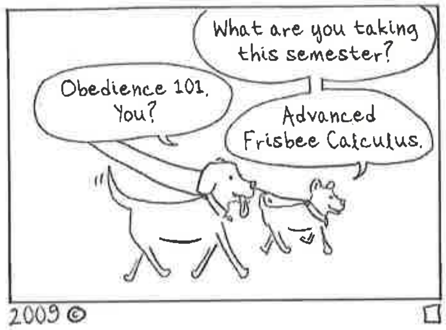
KEY

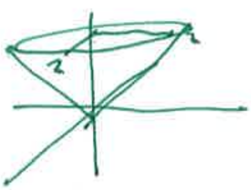
Math 126: Quiz the Eighth
April 21, 2017

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Convert (but don't evaluate) the following integrals

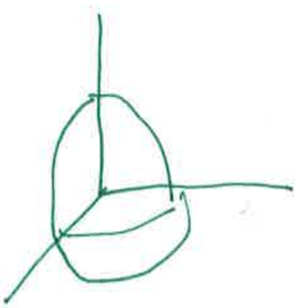
(a) $\displaystyle\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2} x+y \; dz\; dy\; dx$ to cylindrical coordinates.



$$\int_0^{2\pi}\int_0^{2}\int_r^{2} (r\cos\theta + r\sin\theta)\; r \;\; dz\, dr\, d\theta$$

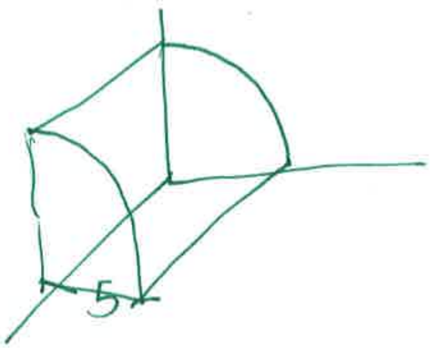
(b) $\displaystyle\int_0^4\int_0^{\sqrt{16-y^2}}\int_{-\sqrt{16-x^2-y^2}}^{\sqrt{16-x^2-y^2}} x+y+z \; dz\; dx\; dy$ to spherical coordinates.

$$\int_0^{\pi}\int_0^{\pi/2}\int_0^{4} (\rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta + \rho\cos\phi)\; \rho^2\sin\phi \; d\rho\, d\theta\, d\phi.$$



sph of radius 4
in 1st octant &
below 1st octant.

2. Evaluate $\iiint_E xy\ dV$ where E is the region in the first octant bound above by $z = 1 - y^2$ and in front by $x = 5$.



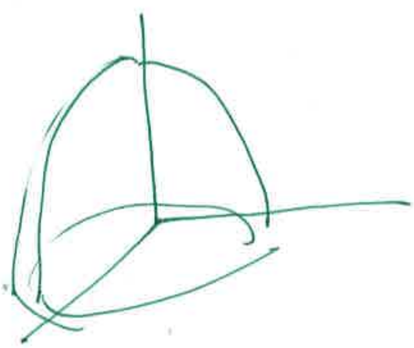
$$\int_0^5 \int_0^1 \int_0^{1-y^2} xy\ dz\, dy\, dx$$

$$= \int_0^5 \int_0^1 x(y - y^3)\, dy\, dx$$

$$= \int_0^5 x\ \frac{y^2}{2} - \frac{y^4}{4} \Big]_0^1 dx = \int_0^5 x\left(\tfrac{1}{2}\right) = \frac{x^2}{4}\Big]_0^5$$

$$= \frac{25}{4}$$

3. Evaluate $\iiint_E \sqrt{x^2 + y^2 + z^2}\ dV$ where E is the top half of a sphere of radius 3 centered at the origin.



$$\int_0^{\pi/2} \int_0^{2\pi} \int_0^3 (\rho)(\rho^2 \sin\phi)\ d\rho\, d\theta\, d\phi$$

$$= \int_0^{\pi/2} \int_0^{2\pi} \frac{\rho^4}{4} \sin\phi \Big]_0^3 d\theta\, d\phi$$

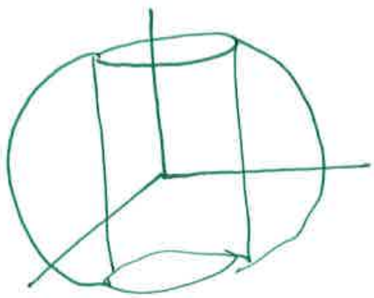
$$= \int_0^{\pi/2} \int_0^{2\pi} \frac{81}{4} \sin\phi\ d\theta\, d\phi$$

$$= \int_0^{\pi/2} \frac{81\pi}{2} \sin\phi\ d\phi$$

$$= \frac{81\pi}{2}(-\cos\phi)\Big]_0^{\pi/2}$$

$$= \frac{81\pi}{2}$$

4. Find the volume outside of the cylinder $x^2 + y^2 = 4$ and inside the sphere $x^2 + y^2 + z^2 = 36$.



$$V_1 = \iiint 1\, dV$$

$$= \int_0^{2\pi}\int_2^6\int_{-\sqrt{36-r^2}}^{\sqrt{36-r^2}} r\, dz\, dr\, d\theta$$

bot sph. $\leq z \leq$ top sph

$2 \leq r \leq 6$

$0 \leq \theta \leq 2\pi$

$$= \int_0^{2\pi}\int_2^6 2r\sqrt{36-r^2}\, dr\, d\theta = \int_0^{2\pi} -\frac{2}{3}\left[36-r^2\right]^{3/2}\Big|_2^6 d\theta$$

$$= \int_0^{2\pi}\left[\frac{2}{3}(32)^{3/2}\right]d\theta = \frac{64\sqrt{2}}{3}\pi$$

5. Find the region E such that $\iiint_E (81 - x^2 - y^2 - z^2)\, dV$ is a maximum. (Hint: think about where the integrand is positive).

We wish to capture all points where

$$81 - x^2 - y^2 - z^2 > 0$$

in our region

thus $81 > x^2 + y^2 + z^2$

and E is the interior of a sphere of radius 9.

6. (Bonus) You may either take 1 point of extra credit for yourself, or none for yourself and give 0.05 points to each of your classmates. Your EC score will be the sum total of points that you take for yourself and points given to you by your peers.