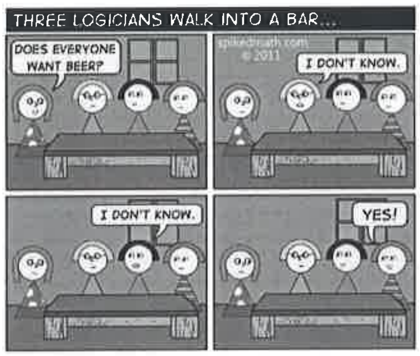
KEY

Math 126: The Penultimate Quiz
April 28, 2017

You have the remainder of the period to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



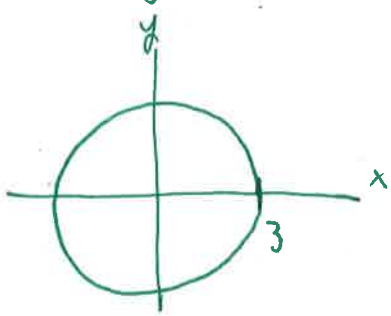
1. Let $\mathbf{r}(t) = \langle 3\cos(t), 3\sin(t), 9\cos^2(t)\rangle$.

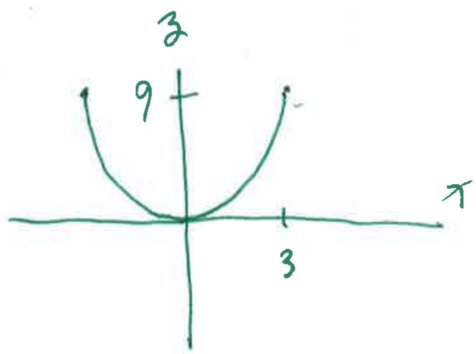
   (a) Plot the view of the curve of $\mathbf{r}(t)$ as viewed in the xy- and xz-planes.

   (b) Find the equation of the tangent line to $\mathbf{r}(t)$ when $t = \frac{\pi}{4}$.

$x^2+y^2=9$ $\qquad$ $z=x^2$

a)





b) $\vec{r}\left(\frac{\pi}{4}\right) = \langle 3\cos\pi/4, 3\sin\pi/4, 9\cos^2\pi/4\rangle$

$= \left\langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, \frac{9}{2}\right\rangle$

$\vec{r}\,'(\pi/4) = \langle -3\sin t, 3\cos t, 18\cos t(-\sin t)\rangle\Big|_{t=\pi/4}$

$= \left\langle -\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, -9\right\rangle$

$\vec{l}(t) = \left\langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, \frac{9}{2}\right\rangle + t\left\langle -\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}, -9\right\rangle$

2. Find a parametrization of the curve that is the intersection between the cylinder $x^2 + z^2 = 16$ and the surface $y = xz$ using

   (a) 'brute force'.

If $x = t$

$z = \pm\sqrt{16 - x^2} = \pm\sqrt{16 - t^2}$

$y = xz = \pm t\sqrt{16 - t^2}$

   (b) Trig identities.

Since $x^2 + z^2 = 16$, set $x = 4\cos t$

$z = 4\sin t$

then $y = 16\sin t\cos t$.

3. The helix $\langle \cos(t), \sin(t), t\rangle$ intersects the curve $\langle 1+t, t^2, t^3\rangle$ at the point (1,0,0).

   (a) Find the t values on each curve where this intersection occurs.

$1 = 1+t$ $\quad$ $1 = \cos t$

$0 = t^2$ $\quad$ $0 = \sin t$

$0 = t^3$ $\quad$ $0 = t$

$t = 0$ $\quad$ $t = 0$

   (b) Find the angle between the curves at the point of intersection (Use the tangent vectors here at the point of intersection). You may leave your answer as an arccosine.

Tangent vectors @ $t=0$

$\langle -\sin t, \cos t, 1\rangle$ $\quad$ $\langle 1, 2t, 3t^2\rangle$

$t=0$ $\quad$ $\hookrightarrow \langle 0, 1, 1\rangle \cdot \langle 1, 0, 0\rangle = 0$

so $\theta = \pi/2$.

4. Find the length of one turn of the helix $\mathbf{r}(t) = \langle 5\cos(\underline{2t}), 12t, 5\sin(\underline{2t})\rangle$. (warning: Watch your bounds...)

One turn $\rightarrow$ $0 \le t \le \pi$

$$\int_0^{\pi} \sqrt{(-10\sin 2t)^2 + (12)^2 + (10\cos 2t)^2}\, dt$$

$$= \int_0^{\pi} \sqrt{100\sin^2 2t + 100\cos^2 2t + 144}\, dt$$

$$= \int_0^{\pi} \sqrt{100+144} = \sqrt{244}\ \pi .$$

5. Find $\int_C x\, ds$ where C is the portion of the parabola $y = 3x^2$ from $(1, 3)$ to $(3, 27)$.

$x = t$

$y = 3t^2$

$1 \le t \le 3$

$ds = \sqrt{(1)^2 + (6t)^2}$

$= \sqrt{1 + 36t^2}$

$$\int_1^3 t\sqrt{1+36t^2}\, dt = \frac{1}{72}\int_{t=1}^{t=3} u^{1/2}\, du$$

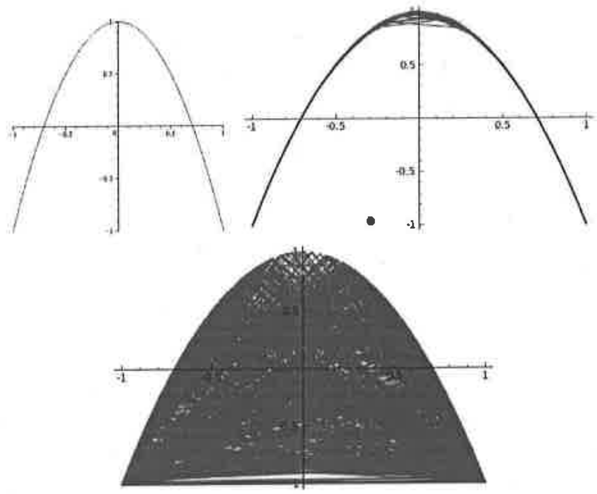
$u = 1 + 36t^2$

$du = 72t\, dt$

$$= \frac{1}{72}\left(\frac{2}{3}\right) u^{3/2} \Big]_{t=1}^{t=3}$$

$$= \frac{1}{108}\left[325^{3/2} - 37^{3/2}\right].$$

6. (Bonus) See the graphs below. They represent the graph of the polar equations $x = \sin(t)$ and $y = \cos(2t)$, which we can show through trig to be the parabola $y = 1 - 2x^2$, but restricted and periodic. The three graphs have increasing bounds on t. Why might we get different pictures for the graphs?



(The ranges on t are $(0, 2\pi)$, $(0, 100\pi)$, and $(0, 500\pi)$ The 'dot' in the second graph is merely a printing error).