Lab The First

Pi and Archimedes

Due Monday, January 29, at 5PM

Archimedes, in his attempt to calculate the ratio of the circumference of a circle to its diameter, used regular polygons of increasing numbers of sides that were either inscribed in or circumscribed about the circle. In this lab you will be carrying out similar geometric techniques, and in so doing, approximating π.

1 Plotting Polygons

In this section, you will be using Maple to plot regular polygons that are inscribed and circumscribed about the unit circle centered at the origin. Maple plots a polygon as a sequence of vertices using the following code

```
with(plots):

inngon := n - > [seq([cos(2*Pi*i/n), sin(2*Pi*i/n)], i = 1..n)]:

display(polygonplot(inngon(8)), color=BLUE);
```

The code invokes the *plots* package, then creates a command *inngon* which takes a value of n and returns the n-gon with vertices at $(x, y) = (\cos(\frac{2\pi i}{n}), \sin(\frac{2\pi i}{n}))$. It then runs the *polygonplot* command on the set of vertices created when $n = 8$. The code also has a command for coloring the polygon blue.

Question 1 *Explain, in detail, why this code gives the vertices of a regular n-gon inscribed in the unit circle centered at the origin. (Hint: Think 'radial angles')*

Problem 1 *Amend the code to create a command* circumngon *that gives the vertices of a regular n-gon circumscribed around the unit circle centered at the origin. You will, of course, first need to determine a formula for these vertices.*

You may plot multiple polygons at the same time by listing them sequentially, as in the following:

```
display(polygonplot(inngon(8)), polygonplot(circumngon(8)), color = white)
```

You can even throw ordinary plots into your polygon plots as well.

Problem 2 *Plot the circumscribed and inscribed n-gons on the same axes as the unit circle for* $n = 4, 8, 16$. *You should have three separate plots.*

2 Calculating Areas and Lengths

You are ready to begin some calculations on your polygons. We'll begin with areas.

Question 2 *What regular dissection might give the best way to calculate the area of this polygon?*

Problem 3 *Determine a formula for the areas of* inngon(n) *and* circumngon(n). *Create a table of these values for* $n = 4, 6, 8, 16, 32$ *and* 64. *What should happen to the respective values as* n *increases?*

Problem 4 *Use calculus to justify what you've observed in the data.*

And now for lengths...

Question 3 *Why is the length of a side of the inscribed n-gon equal to* $2\sin(\frac{\pi}{n})$*?*

The next section involves some heavy trig. I suggest you review your formulas for double angles ($\sin^2(\theta) = \frac{1-\cos(2\theta)}{2}$ and $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$), and stretch a bit.

Problem 5 *What is the perimeter of the square inscribed in the unit circle with its vertices at* $(1, 0), (0, 1), (-1, 0)$ *and* $(0, -1)$*? Justify this using the answer to Question 3.*

Problem 6 *What is the perimeter of the octagon* inngon(8)*? For what value of* θ *do you need to compute* $\sin(\theta)$ *and* $\cos(\theta)$ *in order to compute this?*

Problem 7 *Compute 'clean' formulas for the perimeter of the 16-gon, 32-gon, and 64-gon. You should be hitting on a lovely pattern involving nested square roots. For an extra touch, you might justify your formula using Calculus.*

Once you've finished working your way through the mathematics, research a bit of historical background on Archimedes to include in your report.

Should you choose to use color graphics in your report, please submit an electronic as well as a hard copy of your report. (E-copies can be sent to me at *balofba@whitman.edu*)