Divisibility Problems

1. (1983 Putnam A-1) How many integers divide at least one of 10^{40} and 20^{30}?

2. (1985 Putnam A-4) Define a sequence $\{a_i\}$ by $a_1 = 3$ and $a_{i+1} = 3^{a_i}$ for $i \geq 1$. Which integers between 00 and 99 inclusive occur as the last two digits in the decimal expansion of infinitely many a_i?

3. Prove that no partial sum of the harmonic series $\sum_{k=1}^{n} \frac{1}{k}$ can be an integer for $n > 1$.

4. Find a ten digit number $abcdefghij$ that uses the digits 0 through 9 inclusive exactly once, such that the one digit number a is divisible by 1, the two digit number ab is divisible by 2, the three digit number abc is divisible by three, and so on, with the 10 digit number $abcdefghij$ divisible by 10.

5. Let N be an integer. Prove that there is some integer multiple of N whose digits are all 0's or 1's.

6. You want to color the integers from 1 to 100 so that no number divides another number of the same color. What is the minimum number of colors you must use?