The Binomial Coefficient

1. See the relevant problems in Yaglom and Yaglom as an introduction.

2. For which values n is $\binom{2n}{n}$ odd?

3. (Zeitz, 'The Art and Craft of Problem Solving') We are given n points around a circle and the chords connecting each pair of points is drawn. If no three chords go through the same point, how many intersections are there between chords?

4. (1996, [A–3]) Suppose that each of 20 students has made a choice of anywhere from 0 to 6 courses from a total of 6 courses offered. Prove or disprove: there are 5 students and 2 courses such that all 5 have chosen both courses or all 5 have chosen neither course.

5. (1992, [B-2]) For nonnegative integers n and k, define $Q(n,k)$ to be the coefficient of x^k in the expansion of $(1 + x + x^2 + x^3)^n$. Prove that

$$Q(n,k) = \sum_{j=0}^{k} \binom{n}{j}\binom{n}{k-2j}$$

6. Explain, combinatorially, the 'hockey stick' identity:

$$\binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \ldots + \binom{f}{r} = \binom{f+1}{r+1}$$