## Probability Problems

1. (a) Players $A$ and $B$ play the following game by tossing a coin: If two heads come up in a row, $A$ wins. If two tails come up in a row, $B$ wins. What is the probability that $A$ wins?
(b) Same set up as in (a), but now $A$ wins if three heads are tossed, and $B$ wins if two heads are followed by a tail.
2. (Probabilities and Bridge)
(a) Your two opponents have 6 hearts between their two hands. Is it more likely that each has three, or that one has four and the other two?
(b) If they have 5 hearts between the two hands, is it more likely that they split 3-2, or not?
(c) (The Principle of Restricted Choice) Suppose that your opponents have 4 hearts including the Queen and the Jack (the Queen ranks just above the Jack). If one opponent wins a trick with the Queen, are they more likely, equally likely, or less likely to hold the Jack?
3. Suppose I pass back a set of $n$ exams to $n$ students at random. What is the probability that any given student gets their own exam back. What is the expected number of students that get their own exam back?
4. We have a box of $N+1$ bags of $N$ candies each. In the first bag, all candies are orange. In the second, $N-1$ candies are orange, and one is lemon. In the third bag, $N-2$ candies are orange, and two are lemon, and so on. We take one bag at random and try the candy, which turns out to be lemon flavored. We then draw a second candy from the same bag. What is the probability that it is also lemon flavored?
5. In the border of a perfectly circular piece of wood, we choose $n$ points at random to place legs and make a table. What is the probability that the table will stand without falling?
6. (1993 Putnam B-2) Consider the following game played with a deck of $2 n$ cards numbered from 1 to $2 n$. The deck is randomly shuffled and $n$ cards are dealt to each of two players. Beginning with $A$, the players take turns discarding one of their remaining cards and announcing its number. The game ends as soon as the sum of the numbers on the discarded cards is divisible by $2 n+1$. The last person to discard wins the game. Assuming optimal strategy by both $A$ and $B$, what is the probability that $A$ wins?
