Geometry Problems

- 1. (1996 Putnam A1) Find the least number A such that for any two squares of combined area 1, a rectangle of area A exists such that the two squares can be packed in the rectangle (without interior overlap). You may assume that the sides of the squares are parallel to the sides of the rectangle.
- 2. (2002 Putnam B2) Consider a polyhedron with at least five faces such that exactly three edges emerge from each of its vertices. Two players play the following game:

Each player, in turn, signs his or her name on a previously unsigned face. The winner is the player who first succeeds in signing three faces that share a common vertex.

Show that the player who signs first will always win by playing as well as possible.

- 3. (2000 Putnam A3) The octagon $P_1P_2P_3P_4P_5P_6P_7P_8$ is inscribed in a circle, with the vertices around the circumference in the given order. Given that the polygon $P_1P_3P_5P_7$ is a square of area 5, and the polygon $P_2P_4P_6P_8$ is a rectangle of area 4, find the maximum possible area of the octagon.
- 4. (1999 Putnam B1) Right triangle ABC has right angle at C and $\angle BAC = \theta$; the point D is chosen on AB so that |AC| = |AD| = 1; the point E is chosen on BC so that $\angle CDE = \theta$. The perpendicular to BC at E meets AB at F. Evaluate $\lim_{\theta \to 0} |EF|$.
- 5. (1998 Putnam A1) A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?