Three Probability Conundra

- 1. (Mild) You are on a game show, where the host gives you the choice of three doors. Behind two of the doors are goats, behind the third is a car. You will choose a door, then the host will open one of the doors you did not choose and reveal a goat. You will then be given the chance to switch to the other unopened door. Should you switch, should you stay, or should it matter? (The host knows in advance where the car is located, and, if able to choose between two goats after you've picked your door, will do so randomly).
- 2. (Medium) You are on the same game show. This time, the host has no advance knowledge of the location of the car. He will choose to open a door that you did not open, and if it reveals the car, you lose automatically. If it reveals the car, you're given the same option of switching or sticking. Should you switch, should you stay, or should it matter?
- 3. (Hot) There are two of you on the show now. Each of you picks a different door. At least one of you has picked a goat, so the host will reveal a goat behind one of your two doors, and send that player home. (As always, if the host has an option of whom to excuse, he will do so randomly). The remaining player will be given the option of sticking or switching. Should they stick, should they switch, or should it matter?
- 4. (Caliente) (From Stan Wagon's Problem of the Week, this week!) Alice and Bob face three doors marked 1, 2, 3. Behind the doors are placed, randomly, a car, a key, and a goat. The couple wins the car if Bob finds the car and Alice finds the key. First Bob (with Alice removed from the scene) will open a door; if the car is not behind it he can open a second door. If he fails to find the car, they lose. If he does find the car, then all doors are closed and Alice gets to open a door in the hope of finding the key and, if not, trying again with a second door. Alice and Bob do not communicate except to make a plan beforehand. What is their best strategy?