Induction Practice

1. Prove the three elementary summation identities:

   (a) $1+2+3\ldots+n=\frac{n(n+1)}{2}$

   (b) $1+4+9+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$

   (c) $1+8+27+\ldots+n^3=(\frac{n(n+1)}{2})^2$

2. Prove that
$$1^2+3^2+\ldots+(2n-1)^2=\frac{n(4n^2-1)}{3}.$$

3. Prove that
$$F_1^2+F_2^2+\ldots+F_n^2=F_n*F_{n+1}$$
   where F_n is the nth Fibonacci Number.

4. Suppose that you have n lines in general position (no two parallel). Into how many regions do they divide the plane?

5. Suppose that you have a group of n chess players who play one another in a tournament. Each person plays each other person exactly once and there are no draws. Prove that we can line up all the players such that each person has beaten the person on their immediate right.

6. (2003 Putnam Exam: A1) Let n be a fixed positive integer. How many ways are there to write n as a sum of positive integers, $n=a_1+a_2+\cdots+a_k$, with k an arbitrary positive integer and $a_1\le a_2\le\cdots\le a_k\le a_1+1$? For example, with $n=4$ there are four ways: 4, 2+2, 1+1+2, 1+1+1+1.

7. You have coins $C_1,C_2,\ldots,C_n$. For each k, C_k is biased so that, when tossed, it has probability $\frac{1}{2k+1}$ of falling heads. If the n coins are tossed, what is the probability that the number of heads is odd? Express the answer as a rational function of n. (from an old Putnam Exam)