The Pigeon Hole Principle

1. (a) How many cards must you draw (without replacement) from a standard deck of 52 cards until you are guaranteed two of the same suit?
   (b) How many cards must you draw (without replacement) from a standard deck of 52 cards until you are guaranteed two of the same rank?
   (c) How many people must be in the room until you can guarantee that there are two with the same birthday?
   (d) How many times must you flip a coin until you are guaranteed at least one head?
2. Prove that there are at least two students at Whitman that know the same number of Whitman students.
3. Suppose that there are 17 people at a party. Prove that there are either three who like each other, three who hate each other, or three who don't know each other.
4. Suppose that you have 100 integers (not necessarily distinct). Prove that some subset of them has a sum that is divisible by 100.
5. Prove that any collection of 31 distinct integers between 1 and 60 has the property that one member of the set divides another.
6. Thirty members of the Cannibal Club had a joint dinner. After the dinner, it became known that among every six members of the club, one ate another one. Prove that there are at least 6 members of the club which are inside one another (the first member is inside the second one, the second member is inside the third one and so on).[1]
7. (1996 Putnam A3) Suppose that each of 20 students has made a choice of anywhere from 0 to 6 courses from a total of 6 courses offered. Prove or disprove: there are 5 students and 2 courses such that all 5 have chosen both courses or all 5 have chosen neither course.
8. (2000 Putnam Problem B6) Let B be a set of more than $2^{n+1}/n$ distinct points of the form $(\pm 1, \pm 1, \ldots, \pm 1)$ in n-dimensional space with $n \geq 3$. Show that there are three points in B which are the vertices of an equilateral triangle.

[1] Lends new meaning to the old joke "Why is six afraid of seven?"