KEY

Math 300: Quiz the First

9.11.08

This exam is closed book and closed notes. You may not use a calculator. You have until 5 minutes before the hour to finish the in-class portion. The take-home portion is due Tuesday at the beginning of class.

1. True or False. Give a brief justification in each case.

   (a) If a system of equations has at least two solutions, then it has infinitely many solutions.

   True. Systems have 0, 1, or inf. many solutions

   (b) In performing a row-reduction, you may multiply a row by any scalar.

   False. Scalar must be nonzero

   (c) A matrix might have more than one row-reduced echelon form.

   False. The RREF is unique

   (d) The zero vector is in the span of any set of vectors.

   True $\vec{0} = 0\vec{v}_1 + 0\vec{v}_2 + \cdots + 0\vec{v}_p$

   (e) For every vector $\mathbf{x}$ in $\mathbb{R}^m$, there is a vector $\mathbf{y}$ in $\mathbb{R}^m$ such that $\mathbf{x} + \mathbf{y} = \mathbf{0}$.

   True $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}$ $y = \begin{bmatrix} -x_1 \\ -x_2 \\ \vdots \\ -x_m \end{bmatrix}$

2. Solve the following system of equations:

$$\begin{aligned} x_1 - x_2 - x_3 &= 6 && (1) \\ 2x_1 + x_2 - 3x_3 &= 2 && (2) \\ x_1 - 4x_3 &= 0 && (3) \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 2 & 1 & -3 & 2 \\ 1 & 0 & -4 & 0 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & 3 & -1 & -10 \\ 0 & 1 & -3 & -6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & 1 & -3 & -6 \\ 0 & 3 & -1 & -10 \end{array}\right]$$

$$\xrightarrow{R_3 - 3R_2} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 8 & 8 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -1 & -1 & 6 \\ 0 & 1 & -3 & -6 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{cccc} 1 & -1 & 0 & 7 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$x_1 = 4, \; x_2 = -3, \; x_3 = 1$$

3. Write $\begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$

from above

$$\begin{bmatrix} 6 \\ 2 \\ 0 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + (-3) \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + 1 \begin{bmatrix} -1 \\ -3 \\ -4 \end{bmatrix}$$

4. (a) Suppose that you have a system of equations with more equations than unknowns. Might it have zero, one, or infinitely many solutions? Explain for all that apply.

More equations than unknowns overdetermined but might have

$x_1 + x_2 = 2$ $x_1 + 2x_2 = 3$ $2x_2 + 3x_3 = 4$	$\cancel{x_1 + 2x_2}$ $x_1 + x_2 = 2$ $x_1 + 2x_2 = 3$ $2x_2 + 3x_3 = 5$	$x_1 + x_2 = 2$ $2x_1 + 2x_2 = 4$ $3x_1 + 3x_2 = 6$
0	1	∞ many

(b) Suppose that you have a system of equations with as many equations as unknowns. Might it have zero, one, or infinitely many solutions? Explain.

Determined, might be inconsistent, but may have 1 or ∞ many sol'ns

$x_1 + x_2 = 1$ $2x_1 + 2x_2 = 3$	$x_1 + x_2 = 1$ $x_1 + 2x_2 = 3$	$x_1 + x_2 = 1$ $2x_1 + 2x_2 = 2$
0	1	∞ many

(c) Suppose that you have a system of equations with fewer equations than unknowns. Might it have zero, one, or infinitely many solutions? Explain.

Underdetermined won't have exactly one sol'n but may have

$x_1 + x_2 + x_3 = 2$ $2x_1 + 2x_2 + 2x_3 = 5$	$x_1 + x_2 + x_3 = 2$ $2x_1 + 3x_2 + 4x_3 = 5$
0	∞ many