KEY

Math 300: Quiz the First

This exam is closed book and closed notes. You may not use a calculator. You have until 5 minutes before the hour to finish the in-class portion. The take-home portion is due Tuesday at the beginning of class.

1. True or False. Give a brief justification in each case.

   (a) A system with more unknowns than equations has infinitely many solutions.

   False → might not be consistent.

   (b) In performing a row-reduction, you may multiply a row by any scalar.

   False → any nonzero scalar

   (c) A non-zero matrix must have more than one echelon form.

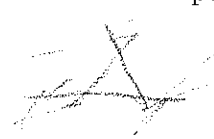
   True → scale any row by any nonzero constant.

   (d) The *existence question* asks whether or not a system of equations has more than one solution.

   False → this is the "uniqueness" question

   (e) Three planes ~~may~~ might not intersect in a point.

   True



2. Solve the following system of equations:

$$\begin{aligned} 2x_1 - 2x_2 + x_3 &= 6 && (1) \\ x_1 + x_2 - x_3 &= -1 && (2) \\ 4x_1 - 2x_3 &= 0 && (3) \end{aligned}$$

$$\left[\begin{array}{ccc|c} 2 & -2 & 1 & 6 \\ 1 & 1 & -1 & -1 \\ 4 & 0 & -2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 2 & -2 & 1 & 6 \\ 4 & 0 & -2 & 0 \end{array}\right]$$

$$\begin{array}{r} \\ \to R_2-2R_1 \\ R_3-4R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & -4 & 3 & 8 \\ 0 & -4 & 2 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & -1 \\ 0 & -4 & 3 & 8 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$x_3 = 4 \,;\; x_2 = \frac{8-3x_3}{-4} = \frac{-4}{-4} = 1$$

$$x_1 = -1 + x_3 - x_2 = -1 + 4 - 1 = 2$$

$$(x_1 = 2,\ x_2 = 1,\ x_3 = 4)$$

3. Write $\begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$

From above

$$\begin{bmatrix} 6 \\ -1 \\ 0 \end{bmatrix} = \underline{2}\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + \underline{1}\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + \underline{4}\begin{bmatrix} 1 \\ -1 \\ -2 \end{bmatrix}$$

4. Convert the following *augmented* matrix to a system of equations.

$$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 3 & 2 \\ 0 & 0 & 1 & -2 & 3 \end{array}\right]$$

and give at least three solutions to the system.

$$x_1 + 2x_2 + 3x_4 = 2$$
$$x_3 - 2x_4 = 3$$

$$\begin{array}{r} \\ \text{free} \rightarrow \\ \\ \text{free} \rightarrow \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 - 2x_2 - 3x_4 \\ x_2 \\ 3 + 2x_4 \\ x_4 \end{bmatrix}$$

Choose any values for x_2, x_4

$$\begin{bmatrix} 2 \\ 0 \\ 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix} \quad \begin{bmatrix} -3 \\ 1 \\ 5 \\ 1 \end{bmatrix}$$

5. (a) Will a system with more equations than unknowns ever have exactly one solution? Explain.

$m \left[\quad \right]$ n

it might, if the system has no free variables.

(each column in the matrix is a pivot)

$m > n$

(b) Will a system with fewer equations than unknowns ever have infinitely many solutions? Explain.

$$\begin{bmatrix} & & & & & \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

yes, and in fact, it will if the system is consistent

(here, we are guaranteed a free variable)

$m < n$