KEY

Math 300: Quiz the Fourth

This exam is closed book and closed notes. You may not use a calculator. You have until 5 minutes before the hour to finish the in-class portion. The take-home portion is due Monday at the beginning of class.

5

1. True or False. Give a brief justification in each case.

   (a) Any line in $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.

   False → must contain the origin

   (b) The set of all polynomials of degree 3 is a vector space.

   False → of degree 3 or lower

   (c) A vector space has a unique 0 vector.

   True → we showed this in class

   (d) A linear transformation is one-to-one if its kernel is empty.

   False 1-1 if its kernel $= \{\vec{0}\}$

   (e) It is difficult to tell if a given vector is in the null space of a matrix A.

   False → multiply your vector by A.

2. Use Cramer's Rule to solve $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & -1 \end{bmatrix}$$

4

and $b = \begin{bmatrix} 4 \\ -2 \\ -3 \end{bmatrix}$

$$\det A = \det\begin{bmatrix} 1 & 0 & 3 \\ 5 & 2 & 3 \\ 1 & 1 & -1 \end{bmatrix} = 1(-2-3)+3(5-2)$$
$$-5+9=4$$

$$A_1 b \to \begin{bmatrix} 4 & 0 & 3 \\ -2 & 2 & 3 \\ -3 & 1 & -1 \end{bmatrix} \leftrightarrow \det A_1(\vec{b}) = 4(-2-3)+3(-2--6)$$
$$= -20+12=-8$$
$$x_1 = \frac{-8}{4} = -2$$

$$A_2 b \to \begin{bmatrix} 1 & 4 & 3 \\ 5 & -2 & 3 \\ 1 & -3 & -1 \end{bmatrix} \to \det A_2(\vec{b}) = 1(2+9)-4(-5-3)+3(-15+2)$$
$$11+32-39=4$$
$$x_2 = \frac{4}{4} = 1$$

$$A_3 b \to \begin{bmatrix} 1 & 0 & 4 \\ 5 & 2 & -2 \\ 1 & 1 & -3 \end{bmatrix} \to \det A_3(\vec{b}) =$$
$$1(-6+2)+4(5-2)$$
$$= -4+12=8$$
$$x_3 = \frac{8}{4} = 2$$

$$x = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

3. Let $V = C(\mathbb{R})$ be the vector space of real-valued continuous functions. Which of the following are subspaces of V? Give a brief justification in each case.

(a) $H = \{f(x) | f(0) = 0\}$

6

$f, g \in H$ Is $\overbrace{f+g \in H}^{C+}$? Is $\overbrace{\lambda f \in H}^{C\times}$? Is $\overbrace{\vec{0} \in H}^{\vec{0}}$?

$C+)$ $(f+g)(0) = f(0) + g(0) = 0 + 0 = 0$ ✓

$C\times)$ $\lambda f(0) = \lambda \cdot 0 = 0$ ✓

$\vec{0})$ $f(x) = 0 \ \forall x \Rightarrow f(0) = 0$ ✓

(b) $H = \{f(x) | f(1) = 0\}$

$C+)$ $f+g(1) = f(1) + g(1) = 0 + 0 = 0$ ✓

$C\times)$ $\lambda f(1) = \lambda(0) = 0$ ✓

$\vec{0})$ $f(x) = 0 \ \forall x \Rightarrow f(0) = 0$ ✓

(c) $H = \{f(x) | f(0) = 1\}$

$C+)$ $f+g(0) = f(0) + g(0) = 1 + 1 = 2 \neq 1$ so $\boxed{\text{NO}}$

(c) is not a subspace.

4. Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 2 & 5 & 1 & 4 \\ 2 & 6 & 0 & 2 & 1 & 3 \\ -1 & -3 & 2 & 3 & 0 & 1 \\ 3 & 9 & 5 & 13 & 1 & 9 \end{bmatrix}$$

6

and its row reduced echelon form

$$R = \begin{bmatrix} 1 & 3 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Give two nonzero vectors that are in col(A) that are not themselves columns of A.

$2C_1 = \begin{bmatrix} 2 \\ 4 \\ -2 \\ 6 \end{bmatrix}$ $\quad C_1 + C_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 8 \end{bmatrix}$ both work

any combinations of columns work!

(b) Determine if $\begin{bmatrix} -5 \\ 2 \\ -3 \\ 2 \\ -1 \\ 1 \end{bmatrix}$ is in nul(A).

$$\begin{bmatrix} 1 \; 3 \; 2 \; 5 \; 1 \; 4 \\ \\ \\ \\ \end{bmatrix} \begin{bmatrix} -5 \\ 2 \\ -3 \\ 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -5+6-6+10-1+4 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} 8 \\ \bullet \\ \bullet \\ ? \end{bmatrix} \neq \vec{0}$$ so NO, it is not.

(c) Is Col(A)=$\mathbb{R}^4$? Why or why not? Let $T(\vec{x}) = A\vec{x}$

No. A does not have a pivot in each Row $\Rightarrow$ $T(\vec{x})$ is not onto

(d) Is Nul(A)={$\mathbf{0}$}? Why or why not?

No. A doesn't have a pivot in each column $\Rightarrow$ $T(\vec{x})$ is not 1-1

(Note: Be explicit about your methods on this one!!)