KEY

Math 349: Quiz the First

January 30, 2014

1. Ariel, Belle, Cinderella, Diana, and Ella want to line up for a group photo. How many arrangements are possible if

   (a) Ella will not be on either end?

Ella 3 places, others $4!$ places $= 3 \cdot 4! = 72$

Alt: total $5!$ places $-\ 4!$ Ella on left $-\ 4!$ Ella on right $= 5! - 4! - 4! = 4!(5-1-1) = 3\,4!$

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   (b) There must be exactly one person between Ariel and Belle.

$$\underset{\substack{\text{choices for}\\ \text{person between}\\ A\ \&\ B}}{3} \cdot \underset{\substack{\text{arrangements}\\ \text{for}\\ A\ \&\ B}}{2} \cdot \underset{\substack{\text{treat } A \times B\\ \text{as a unit,}\\ \text{arranging 3 objects}}}{3!} = 36$$

2. Two gamblers are playing a game multiple times, with each player equally likely to win either game. The first person to win 6 games wins the \$100 stake. If the game is interrupted with the score 4-2, what is a fair division of the stakes?

play out the remaining (at most) 5 games → 32 ways.

The player ahead will win $\binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$ of these series

$= 10 + 10 + 5 + 1 = 26$ series

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So we should divide the stakes

$$\frac{26}{32} \cdot 100, \quad \frac{6}{32} \cdot 100.$$

3. In an experiment, a student will toss a coin and throw a fair six-sided die, and record the outcomes.

   (a) How big is the Sample Space?

   Toss A Coin · Throw a Die

   $2 \cdot 6 = 12$ elements.

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   (b) Describe the event E: The roll of the die is at least 5.

   $E = \{(H,5), (T,5), (H,6), (T,6)\}$

4. Let A be an event. Prove directly from the three axioms of Probability that $P(A^c) = 1 - P(A)$.

A and A^c are mutually exclusive,

hence

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$$P(A \cup A^c) = P(A) + P(A^c)$$
$$P(S) = P(A) + P(A^c)$$
$$1 = P(A) + P(A^c)$$
so $P(A^c) = 1 - P(A)$.

5. (a) Suppose that in a city, 53% of the population is female, and that 15% are unemployed males. If we select a person at random what is the probability that the person is an employed male?

$$\begin{array}{ll} 47\% & \text{male} \\ -\ 15\% & \text{unemployed males} \\ \hline 32\% & \text{employed males.} \end{array}$$

$P = .32.$

(b) Suppose further that the unemployment rate in the city is 22%. What is the probability that the person selected is an unemployed female?

unemployment rate

$$22\% = \underset{\text{male}}{15\%} + \underset{\text{Female.}}{7\%}$$

$P = .07.$

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(c) What is the probability that the person selected is employed or a female or both?

$$P(\underset{\substack{\uparrow \\ \text{employed}}}{E} \cup \underset{\substack{\uparrow \\ \text{female}}}{F}) = P(E) + P(F) - P(EF)$$

$$.78 + .53 - .46$$

$$1.31 - .46 = \underline{\underline{.85}}$$

6. A person claims that there is a 20% chance that the sun will shine tomorrow, a 40% chance that it will shine the next day, a 15% chance that it will shine both days, and a 50% chance that it will shine on at least one day. Explain to them (politely!) why they must be wrong.

$T \rightarrow$ sun shines tomorrow

$N \rightarrow$ sun shines the next day

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$P(T \cup N) = P(T) + P(N) - P(TN)$

$.50 \neq .2 + .4 - .15 = .45$

But....

So, in fact, its only a 45% chance that the sun will shine in the next two days

But I appreciate your optimism

7. Pretend to toss a coin five times and record the list of heads and tails.

H T H H T