

Math 386, Supplemental Exercises
Due February 11, 2013, in class

1. For each field extension below, find $\Gamma(L : K)$
 - (a) $L = \mathbb{Q}(\alpha)$, $K = \mathbb{Q}$, where $\alpha = \sqrt[5]{2}$
 - (b) $L = \mathbb{Q}(\alpha)$, $K = \mathbb{Q}$, where $\alpha = \zeta_3$
 - (c) $L = \mathbb{Q}(\alpha, \beta)$, $K = \mathbb{Q}(\alpha)$, where $\alpha = \zeta_3, \beta = \sqrt[3]{2}$.
 - (d) $L = \mathbb{Q}(\alpha)$, $K = \mathbb{Q}$, where $\alpha = \sqrt[4]{2}$ (harder than it looks...)
2. Let $\alpha \in \mathbb{C}$ be algebraic over \mathbb{Q} and let $r \in \mathbb{Q}$. Prove that α^r is algebraic over \mathbb{Q} .
3. Is π algebraic over $\mathbb{Q}(\pi^3)$? Why or why not?
4. Suppose that $E : F$ is an extension and that $[E : F] = p$, a prime. Prove that, for all $a \in E$, $F(a) = F$ or $F(a) = E$.