Interference, Complementarity, Entanglement and all that Jazz

Mark Beck
Dept. of Physics, Whitman College

With lots of help from:

Faculty: Robert Davies (Seattle U)
Students: Ashifi Gogo, William Snyder, Jeremy Thorn (U of O), Matthew Neel (OSU), Vinsunt Donato (OSU)
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Quantum Mechanics

Quantum information is changing how we think about quantum systems.
- Convey this to students
Many experiments involve photons
- Doable by undergraduates

Project Goals

1) Develop a series of advanced undergraduate laboratories exploring modern aspects of quantum mechanics
   - Study the properties of individual photons
2) Develop course materials that take advantage of these labs
   - Use photon polarization as an example 2-dimensional quantum system
**Experiment Proving Photons Exist**

1) Should be conceptually simple
2) Should display the "granular" nature of individual photons
3) Necessary to treat the field quantum mechanically
   - Not explainable using classical waves

**Proving Photons Exist**

**Photoelectric Effect?**
- Satisfies criteria 1) & 2)
  - detector "clicks" are granular
- Does NOT satisfy criterion 3)
  - Does not require photons (i.e. a quantum field) for its explanation
  - Can be explained using a semiclassical theory (detector atoms quantized, field is a classical wave)

**Grangier Experiment**

**Single Photon on a Beamsplitter**

If a single photon is incident on a beamsplitter, what do we know about "clicks" at output detectors?
- Only one detector will fire
- No coincidence detections

"...a single photon can only be detected once!"
- Grangier et al.
Quantify:

The degree of second-order coherence

Single Photon on a Beamsplitter

Quantify:

\[ g^{(2)}(0) = \frac{\langle I_T I_R \rangle}{\langle I_T \rangle \langle I_R \rangle} \]

\[ = \frac{P_T}{P_T P_R} \]

\[ P_R = 0 \]

\[ \therefore g^{(2)}(0) = 0 \quad \text{(for a single photon input)} \]

The degree of second-order coherence

Classical Wave on a Beamsplitter

Quantify:

\[ g^{(2)}(0) = \frac{\langle I_T I_R \rangle}{\langle I_T \rangle \langle I_R \rangle} \]

\[ = \frac{P_T}{P_T P_R} \]

\[ I_T = T I, \quad I_R = R I, \quad T + R = 1 \]

\[ g^{(2)}(0) = \frac{\langle I_T^2 \rangle}{\langle I_T \rangle} \]

\[ \langle I_T^2 \rangle \geq \langle I_T \rangle \quad \text{(Cauchy-Schwartz inequality)} \]

\[ \therefore g^{(2)}(0) \geq 1 \quad \text{(for a classical wave)} \]

Positive Correlations

Fluctuating input wave simply splits equally at a

beamsplitter

\[ T \quad \text{and} \quad R \quad \text{are most likely to click at the same time} \]

\[ \text{• Opposite behavior of a single photon} \]
Distinguishing Classical and Quantum Fields

Classical waves: \( g^{(2)}(0) \geq 1 \)

Therefore, any field with \( g^{(2)}(0) < 1 \) cannot be described classically, and is inherently quantum mechanical.

Single photon state: \( g^{(2)}(0) = 0 \)

Making a Single Photon State

Spontaneous parametric downconversion

- One photon converted into two

\[ \omega_p = \omega_1 + \omega_2 \]

Nominally:

\[ \omega_1 \sim \omega_2 \sim \omega_p / 2 \]

Making a Single Photon State

Spontaneous parametric downconversion

- One photon converted into two
- Photons always come in pairs

\[ \omega_1 \sim \omega_2 \sim \omega_p / 2 \]

You know you have a photon in the other beam

In 1986 Grangier et al. used a cascade decay in Ca as a photon pair source.
Look for coincidences between $T$ and $R$, conditioned on a detection at $G$.

Everything is conditioned on a detection at $G$:

$$P_{TTE} = \frac{N_{TT}}{N_{G}}$$

$$g^{(2)}(0) = \frac{P_{TT}}{P_{T}P_{T}}$$

$$P_{TTE} = \frac{N_{TT}}{N_{G}}$$

$$g^{(2)}(0) = \frac{N_{TT}N_{G}}{N_{TT}N_{TT}}$$

Our Experiment

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More Details

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Experimental Setup

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**Experimental Setup**

**Collection Optics**

**Results**

<table>
<thead>
<tr>
<th>Integration time per pt.</th>
<th>Number of pts.</th>
<th>Total acq. time</th>
<th>$g^{(2)}(0)$</th>
<th>St. dev. of $g^{(2)}(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.7 s</td>
<td>110</td>
<td>~ 5 min.</td>
<td>0.0188</td>
<td>0.0067</td>
</tr>
<tr>
<td>5.4 s</td>
<td>108</td>
<td>~ 10 min.</td>
<td>0.0180</td>
<td>0.0041</td>
</tr>
<tr>
<td>11.7 s</td>
<td>103</td>
<td>~ 20 min.</td>
<td>0.0191</td>
<td>0.0035</td>
</tr>
<tr>
<td>23.4 s</td>
<td>100</td>
<td>~ 40 min.</td>
<td>0.0177</td>
<td>0.0026</td>
</tr>
</tbody>
</table>

In 5 minutes of counting we violate the classical inequality $g^{(2)}(0) \geq 1$ by 146 standard deviations.
Why not 0?

Perfect single photons have $g^{(2)}(0) = 0$.
- i.e., we expect no coincidences between $T$ and $R$

Why do we measure $g^{(2)}(0) = 0.0177 \pm 0.0026$?
- Accidental coincidences
  - Due to finite coincidence window (2.5 ns)

Expected accidental coincidence rate explains difference from 0.

Single Photon Interference

- Easy to align
  - equal pathlengths
- EXTREMELY stable

Polarization Interferometer
Results

Raw Counts ($V=88\%$)

Coincidence Counts, Single Photons ($V=89\%$)

Simultaneously displays wave-like (interference) and particle-like ($g^{(2)}(0)<1$) behavior.

Wave-Particle Complementarity
Wave-Particle Complementarity

Exits this way

Or this way

Wave-Particle Complementarity

Never both ways

Not even if phase of interferometer is adjusted so that on average the photon goes each way half the time.
**Wave-Particle Complementarity**

- Goes both ways at first beamsplitter
- Goes one way at second beamsplitter

Wave-like behavior inside the interferometer
Particle-like behavior outside the interferometer

**Results**

- Raw Counts ($V=88\%$)

- Coincidence Counts, Single Photons ($V=89\%$)

Simultaneously displays wave-like (interference) and particle-like ($g^{(2)}(0)<1$) behavior.

**Results**

Raw Counts

$L_c=12\mu m$ ($\Delta \lambda =57\text{nm}$)

$L_c = \frac{c}{\Delta \nu} = \frac{\lambda^2}{\Delta \lambda}$

Coincidence Counts, Single Photons

$L_c=70\mu m$ ($\Delta \lambda =10\text{nm}$)

Interference filter in gate beam ($\Delta \lambda =10\text{nm}$)
Frequency Entanglement

Inserting interference filter to decrease bandwidth of this beam

Increases the coherence length of coincident photons in this beam

Entanglement

Frequencies of the two beams are entangled

\[ \omega_p = \omega_G + \omega_I \]

(\( \omega_p \)) frequency of pump (blue) beam

\( \omega_G, \omega_I \) frequencies of gate and interferometer beams

In coincidence, narrowing the distribution of \( \omega_G \) narrows the distribution of \( \omega_I \)

Present Source

\[ \psi_{\omega} = |\uparrow, \downarrow\rangle \]
Polarization Entangled Source

2 crystals
Axes rotated by 90°

\[ |\psi_{\lambda2}\rangle = \frac{1}{\sqrt{2}} (|\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle) \]

Quantum Eraser

Horizontal, Vertical
**Results**

- Takes one path
  - Have which-path info
  - No interference

How do we see interference?
- Must take both paths
- Erase which-path info
Quantum Eraser

Entangled in *Any* Basis

\[ |\psi_{\pm}\rangle = \frac{1}{\sqrt{2}} (|\uparrow,\downarrow\rangle + |\leftrightarrow,\leftrightarrow\rangle) \]

\[ = \frac{1}{\sqrt{2}} (|\kappa\rangle + |\chi\rangle) \]
± 45°

Photon takes both paths —
Interference

Results
Takes both paths
• Erased which-path info
• Interference
Raw counts (not coincidence)
• No interference
• Why?

PBS
G \left( |\psi\rangle \right)

\lambda/2

\psi \rightarrow |\psi^\prime, \psi^\prime\rangle

\psi_{\text{raw}}
collide
|\psi^\prime, \psi^\prime\rangle

± 45°
No Coincidence Detection

Detection at $T$ does NOT yield which-path info.

Why no interference?

COULD Get Info

COULD IN PRINCIPLE know which way the photon went—
No interference
Results

Each polarization makes its own interference pattern
  - Out of phase

Add $G$ and $G'$ coincidences
  - No interference
  - Why?
Again, we’re not using gate polarization info
  - Could do a different measurement that provides which-path information
Quantum Eraser

Interference
- Not having which-path information is not good enough
- The fact that which-path information is available in principle is enough to destroy interference

Only way to erase in principle information is to explicitly perform a measurement that erases it.

Conclusions

We have performed the following experiments
- Proof of the existence of photons
- Single-photon interference
- Quantum eraser
  - A classical mixed state can mimic certain aspects of the eraser behavior
- Test of Bell inequality
  - $S=2.467\pm0.015$
  - Violates $S<2$ by 30 standard deviations

All experiments have been performed by undergraduates, and are suitable for an undergraduate laboratory

Whole Table

http://www.whitman.edu/~beckmk/QM/