# Quantum mysteries tested: An experiment implementing Hardy's test of local realism

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We have performed a test of local realism using entangled photons produced by spontaneous parametric downconversion. This experimental test is based on an idea originally proposed by Hardy for a test of local realism without inequalities, although our experiment actually measures an inequality (as any experiment must). We find a more-than-70 standard deviation violation of the predictions of local realism. The experimental effort required for this test is essentially the same as that required for a test of a Bell inequality. However, this test is based on concepts that are easier to understand and more compelling than those behind the original Bell inequality. 

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## I. INTRODUCTION

Over the years numerous papers by Mermin and others have explored with ever increasing clarity and simplicity some of the mysteries of quantum mechanics.[1–5] By mystery we mean the tendency of quantum mechanics to predict outcomes that contradict our deeply ingrained classical notions. In particular, quantum mechanics violates local realism. Locality here means that the results of measurements in one location should not affect the results of measurements in another location if there is no causal relationship between the measurements. Reality refers to the idea that we should be able to assign definite values to physically measurable quantities prior to their actual measurement. By saying quantum mechanics violates local realism, we mean that we are forced to give up either locality or reality to explain quantum mechanical predictions.

The idea that quantum mechanics violates local realism dates to the ideas of Einstein, Podolsky, and Rosen (EPR).[6] It was Bell who showed that it is possible to perform an experiment to prove that quantum mechanics violates local realism.[7] He did so by deriving an inequality that must be satisfied by any local realistic system and then showing that quantum mechanics could violate this inequality. Since Bell's original work other similar inequalities have been derived, which are often collectively referred to as Bell inequalities. There have been numerous experimental tests of Bell inequalities, with nearly all of them confirming the predictions of quantum mechanics (see Ref. 8 and the references therein). Dehlinger and Mitchell performed a test of a Bell inequality in an undergraduate laboratory,[8] and we and others have implemented this experiment in undergraduate laboratories since then.

Greenberger, Horne, and Zeilinger went beyond Bell inequalities by showing that it is, in principle, possible to perform an "all or nothing" test of local realism.[2,9] This result was an advance because the argument by Bell was essentially statistical in nature—classical physics predicts certain results occur with one probability, while quantum mechanics predicts the same results occur with a different probability. In an all-or-nothing test, classical mechanics predicts a certain outcome will always happen, while quantum mechanics predicts it never happens. All-or-nothing tests are significantly more difficult experimentally, because they usually involve the use of three entangled particles in contrast to the two particles needed for tests of Bell inequalities. Nevertheless, the experiment has been performed and agrees with quantum mechanical predictions.[10]

In 1993 Hardy derived what Mermin has referred to as "the best version of Bell's theorem."[11,12] Hardy conceived of a system of two particles in which local realism predicts a certain outcome never occurs, while quantum mechanics predicts that it sometimes occurs. In principle, the observation of a single occurrence of this event is enough to demonstrate that quantum mechanics violates local realism. In practice, a real experiment with an imperfect apparatus must measure an inequality; we discuss this point further in the following. Experiments testing local realism using Hardy's ideas have been performed and agree with quantum mechanics.[13–15]

Because both experiments involve similar inequalities, it might be thought that there is no advantage to performing Hardy's test of local realism over Bell's test. However, the explanation of Hardy's test is much easier to understand. Although it might take an hour or more to explain Bell's theorem to students in a junior/senior quantum mechanics course (and longer to derive it), it takes about half an hour to explain Hardy's ideas to a nontechnical audience. This difference is a compelling reason to implement Hardy's test of local realism in an undergraduate laboratory.

## II. THEORY

### A. The basic idea

There are many discussions of Hardy's ideas aimed at undergraduate and nontechnical audiences. Each uses a different analogy between the real experimental apparatus and other objects that are more familiar to the reader. The scenarios involve objects such as flashing colored lights,[3] Dutch doors,[16] cakes,[5] and socks.[17] We have our own scenario involving observations made by two students sitting in different classrooms. Because these simplified explanations are available elsewhere,[3,5,16] the explanation we present here involves the measurement of the polarization of two photons, which is what we actually do in our experiment.

Imagine a source that produces pairs of photons that travel in different directions—one goes to Alice and the other goes to Bob, as shown in Fig. 1. For simplicity we will assume that the photons come at regular intervals, and that Alice and Bob can measure them with 100% efficiency. Alice's photon passes through a linear polarizer and then on to a detector;

  

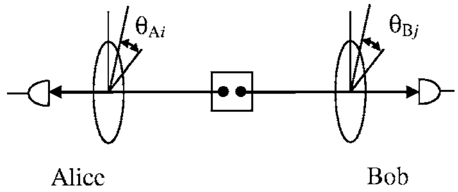


Fig. 1. The gedanken experiment.

she randomly orients her polarizer axis along direction $\theta_{A1}$ or $\theta_{A2}$. If Alice measures a photon at her detector, then she knows it was polarized parallel to the polarizer axis (for example, along $\theta_{A1}$). Bob has an identical polarizer and detector that he uses to perform the same measurement on his photon. The angles that Bob randomly orients his polarizer along are $\theta_{B1}$ and $\theta_{B2}$. Alice and Bob make completely independent measurements and do not communicate during the measurement procedure. Once all the measurements have been performed, they get together to compare their data.

In principle, it could be arranged to make it impossible for Alice's and Bob's measurements to influence each other by ensuring that the time interval between Alice's and Bob's measurements is smaller than the time it would take for light to travel between Alice and Bob (that is, the measurements are separated by a spacelike interval). If this condition is satisfied, we say that Alice and Bob have established strict locality conditions; Alice's and Bob's measurements are determined solely by the interaction of their own particles with their own local detection apparatus.[18] There may still be correlations between the measurements made by Alice and Bob; however, these correlations must be determined when the photons are created at the source (for example, both photons always have the same polarization) and not influenced by measurements performed far away. This lack of influence is essentially the reality assumption—that it makes sense to talk about a photon as having a definite polarization after it leaves the source, but before it is measured.

When Alice and Bob compare their data, they are interested in the probabilities that they will observe certain polarizations. They are interested in the joint probability $P(\theta_{Ai},\theta_{Bj})$ that Alice will measure her photon to have polarization $\theta_{Ai}$ and Bob will measure his photon to have polarization $\theta_{Bj}$. They are also interested in conditional probabilities of the form $P(\theta_{Bj}|\theta_{Ai})$, which is the probability that Bob will measure $\theta_{Bj}$ given that Alice measured $\theta_{Ai}$. After calculating various probabilities from their data, Alice and Bob notice four interesting features, which we will refer to as observations 1–4.

1. Alice sets her polarizer along $\theta_{A1}$ and Bob sets his polarizer along $\theta_{B1}$. In this case they will sometimes both detect photons. In fact, with these polarizer settings the photons are found to have this combination of polarizations about 9% of the time; that is, $P(\theta_{A1},\theta_{B1})\cong 0.09$.
2. Alice sets her polarizer along $\theta_{A1}$ and Bob sets his polarizer along $\theta_{B2}$. If Alice detects a photon, then Bob always detects a photon; that is, $P(\theta_{B2}|\theta_{A1})=1$.
3. Alice sets her polarizer along $\theta_{A2}$ and Bob sets his polarizer along $\theta_{B1}$. If Bob detects a photon, then Alice always detects a photon; that is, $P(\theta_{A2}|\theta_{B1})=1$.

By logical inference, we can use these three observations to make a prediction about what must happen if Alice sets her polarizer along $\theta_{A2}$ and Bob sets his polarizer along $\theta_{B2}$. Let's start by imagining only those situations in which Alice measures her photon to be polarized along $\theta_{A1}$ and Bob measures his to be polarized along $\theta_{B1}$ (by observation 1 we know that this situation is allowed). Suppose that at the last moment Bob changes his mind and instead orients his polarizer along $\theta_{B2}$. What will he find? Because we know Alice will measure a photon polarized along $\theta_{A1}$, by observation 2 we know that Bob must measure his photon to be polarized along $\theta_{B2}$. What if, instead, Bob leaves his polarizer oriented along $\theta_{B1}$, but Alice changes her mind and orients her polarizer along $\theta_{A2}$? Because Bob will measure his photon to be along $\theta_{B1}$, by observation 3 we know that Alice must measure her photon to be polarized along $\theta_{A2}$. What happens if both Alice and Bob change their minds and measure along $\theta_{A2}$ and $\theta_{B2}$? By using similar reasoning, we conclude that Alice and Bob must measure their photons to be polarized along $\theta_{A2}$ and $\theta_{B2}$.

Observations 2 and 3 require that every time Alice and Bob would have measured photons along $\theta_{A1}$ and $\theta_{B1}$, they will instead measure them to be polarized along $\theta_{A2}$ and $\theta_{B2}$ if they choose to orient their polarizers differently. Thus with the reasonable assumption that the properties of the photons produced at the source are independent of the detector settings,[19] Alice and Bob must measure photons polarized along $\theta_{A2}$ and $\theta_{B2}$ at least as often as they measure photons polarized along $\theta_{A1}$ and $\theta_{B1}$.

$$P(\theta_{A2},\theta_{B2}) \geqslant P(\theta_{A1},\theta_{B1}). \tag{1}$$

By observation 1 they must measure photons polarized along $\theta_{A2}$ and $\theta_{B2}$ at least $\sim$9% of the time that they have their polarizers oriented along those axes: $P(\theta_{A2},\theta_{B2})\geqslant \sim 0.09$.

What do Alice and Bob observe when they set their polarizers along $\theta_{A2}$ and $\theta_{B2}$? This question brings us to observation 4;

4. If Alice sets her polarizer along $\theta_{A2}$ and Bob sets his polarizer along $\theta_{B2}$, they never find their photons to be simultaneously polarized in these directions: $P(\theta_{A2},\theta_{B2})=0$.

Clearly, the experimental evidence of observation 4 violates $P(\theta_{A2},\theta_{B2})\geqslant \sim 0.09$. Alice and Bob have been performing an experiment on a quantum mechanical system, and their results contradict the classical reasoning that leads to this inequality. This evidence means that at least one of our assumptions is incorrect. Because the only assumptions we made are those of locality and reality, we find that quantum mechanics violates local realism.[20] Of course, we must also show that quantum mechanics allows for a source of photon pairs that satisfy observations 1–4 ( see Appendix A).

We have stated that Hardy's test of local realism does not require an inequality. If we first perform an experiment that verifies observations 2, 3, and 4, then Eq. (1) forces us to conclude that $P(\theta_{A1},\theta_{B1})=0$. Suppose Alice and Bob set their polarizers along $\theta_{A1}$ and $\theta_{B1}$. If they ever measure simultaneous counts, this conclusion is false, and quantum mechanics is found to violate local realism. In this sense a single measurement, and not an inequality, is needed to violate local realism.

### B. The experimental inequality

The argument in Sec. II A hinges on the fact that certain events happen 100% of the time. What if these events are observed to happen only 98% of the time? Does the entire argument fall apart? We now show that it does not, but it does need to be modified.

Let us reexamine observation 2, which says that if Alice's photon is found to be polarized along $\theta_{A1}$, then Bob's must be found to be polarized along $\theta_{B2}$. If this statement is true, then it will never be the case that Alice finds her photon polarized along $\theta_{A1}$ and Bob finds his photon polarized perpendicular to $\theta_{B2}$. Similar reasoning can be applied to observation 3, and we can rewrite these observations in the form

2′. If Alice sets her polarizer along $\theta_{A1}$ and Bob sets his polarizer along $\theta_{B2}^{\perp}=\theta_{B2}\pm 90°$, Alice and Bob never detect simultaneous photons and $P(\theta_{A1},\theta_{B2}^{\perp})=0$.

3′. If Alice sets her polarizer along $\theta_{A2}^{\perp}$ and Bob sets his polarizer along $\theta_{B1}$, Alice and Bob never detect simultaneous photons and $P(\theta_{A2}^{\perp},\theta_{B1})=0$.

Now the argument goes that observations 2′ and 3′ imply Eq. (1), which is violated by observations 1 and 4. An experimental test of local realism involves measuring four joint probabilities and verifying that they satisfy observations 1, 2′, 3′, and 4.

Verifying observations 2′ and 3′ involves showing that the measured probabilities are equal to zero. Such a verification is impossible experimentally, because a measured probability can never be shown to be equal to zero for two reasons. One is that experimental imperfections (for example, dark counts, accidental coincidences) invariably lead to nonzero probabilities. Indeed, a zero measurement should be viewed with extreme skepticism—it would likely be due to the fact that the detectors are not working![21] The second reason is that just because we make a million measurements and do not see something, does not mean we won't see it on the million and first measurement. In other words, if we perform $N$ measurements, the best that we can say is that the probability is less than $\sim 1/N$. Because of these experimental realities, it is necessary to recast our test of local realism in a form that allows for nonzero probabilities.

Lets suppose that 2% of the time Alice and Bob detect simultaneous photons when Alice sets her polarizer along $\theta_{A1}$ and Bob sets his polarizer along $\theta_{B2}^{\perp}: P(\theta_{A1},\theta_{B2}^{\perp})=0.02$. Thus, it is reasonable to expect that 2% of the time we would obtain the incorrect answer when we try to infer what Alice and Bob will measure with their polarizers set to $\theta_{A2}$ and $\theta_{B2}$. Assume the worst possible scenario in which we obtain the wrong answer 2% of the time. We can no longer say that Alice and Bob will measure $\theta_{A2}$ and $\theta_{B2}$ at least as often as they measure $\theta_{A1}$ and $\theta_{B1}$; it is possible that they will measure $\theta_{A2}$ and $\theta_{B2}$ 2% less often than they will measure $\theta_{A1}$ and $\theta_{B1}$. We must subtract the probability that we are incorrect, $P(\theta_{A1},\theta_{B2}^{\perp})$, from the probability $P(\theta_{A1},\theta_{B1})$. If the probability $P(\theta_{A2}^{\perp},\theta_{B1})$ is nonzero, it must be subtracted as well. The predictions of local realism are no longer given by Eq. (1), but instead become

$$P(\theta_{A2},\theta_{B2}) \geqslant P(\theta_{A1},\theta_{B1}) - P(\theta_{A1},\theta_{B2}^{\perp}) - P(\theta_{A2}^{\perp},\theta_{B1}). \quad (2)$$

Equation (2) is a form of the Bell-Clauser-Horne inequality,[3,15] which must be satisfied by any system that is local and realistic, but which quantum mechanics can violate. We have not derived this inequality, but have merely motivated it; the reader is referred elsewhere for a proof.[3]

Equation (2) is the general form of the Bell-Clause-Horne inequality, which involves four independent angles. In our experiment the angles of interest are the angles $\alpha$ and $\beta$ and their negatives. By assigning $\theta_{A1}=\beta$, $\theta_{B1}=-\beta$, $\theta_{A2}=-\alpha$, and $\theta_{B2}=\alpha$, we can rewrite Eq. (2) as

$$P(-\alpha,\alpha) \geqslant P(\beta,-\beta) - P(\beta,\alpha^{\perp}) - P(-\alpha^{\perp},-\beta). \quad (3)$$

It is convenient to define the quantity $H$ by

$$H = P(\beta,-\beta) - P(\beta,\alpha^{\perp}) - P(-\alpha^{\perp},-\beta) - P(-\alpha,\alpha). \quad (4)$$

The Bell-Clauser-Horne inequality allows us to divide the behavior of $H$ into two regions: $H \leqslant 0$ is consistent with local realism, while $H > 0$ is inconsistent with local realism. The fact that $H > 0$ is allowed by quantum mechanics is shown in Appendix A.

## III. EXPERIMENTS

Our experiment uses polarization entangled photons produced in spontaneous parametric downconversion. In this process a single photon at one frequency, called the pump photon, is converted into two photons of lower frequency in a nonlinear crystal. The lower frequency photons are referred to as the signal and the idler. Energy conservation requires that the energies of the signal and idler photons add up to the energy of the pump photon; in practice this requirement means that the frequencies of the signal and idler photons are approximately half of the frequency (twice the wavelength) of the pump photon.

An experiment to perform Hardy's test of local realism[15] is nearly identical to a test of Bell's inequality.[8,22] The apparatus required is the same; the only differences when performing Hardy's test are (a) the downconversion source is tuned slightly differently to produce photons in a different quantum state; (b) the polarizers in front of the detectors are set to different angles; (c) the data is analyzed to compute the quantity $H$, defined in Eq. (4), rather than a different quantity that is used in tests of Bell's inequality.

### A. Experimental apparatus

Our experimental apparatus is depicted in Fig. 2, and a list of the equipment we use is provided in Appendix B. The source consists of a pair of 0.5-mm-thick BBO crystals, each cut for type-I downconversion (the polarization of the signal and idler photons is perpendicular to the polarization of the pump). They are stacked back to back, with their crystal axes oriented at 90° with respect to each other.[8,15,22] The first crystal converts vertically polarized pump photons into horizontally polarized signal and idler, while the second crystal converts horizontally polarized pump photons into vertically polarized signal and idler. The pump laser is a 50-mW, 405-nm laser diode, so the wavelength of the signal and idler photons is centered about 810 nm. The half-wave plate, $\lambda/2$, and the quartz plate in front of the downconversion crystals are used to adjust the pump polarization and the relative phase between the horizontal and vertical polarizations. The signal and idler photons each make an angle of approximately 3° from the direction of the pump. The signal photons travel to Alice's detection apparatus, and the idler photons travel to Bob's.

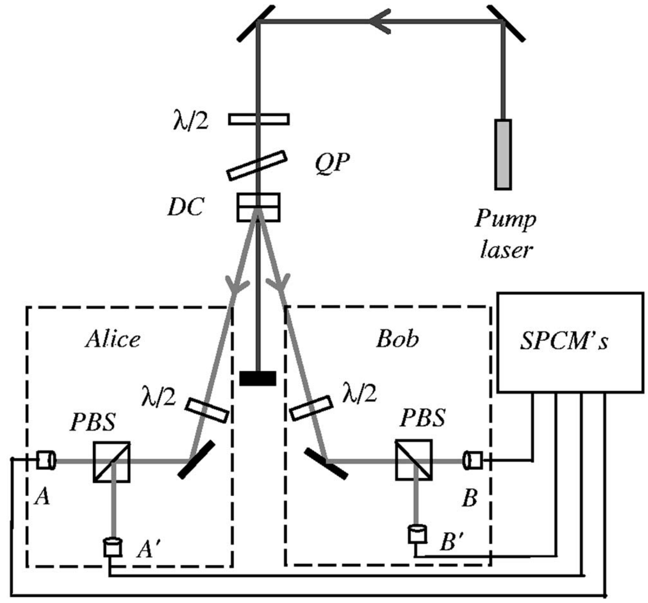


Fig. 2. The experimental arrangement. Here $\lambda/2$ denotes a half-wave plate, QP denotes the quartz plate, DC denotes the downconversion crystals, SPCM denotes a single photon counting module, and PBS denotes a polarizing beam splitter.

The state we wish to produce has the form

$$|\psi\rangle = a|H\rangle_A|H\rangle_B + be^{i\phi}|V\rangle_A|V\rangle_B, \tag{5}$$

where $H$ refers to a horizontally polarized photon, $V$ refers to a vertically polarized photon, $a$ and $b$ are real numbers, $\phi$ is the relative phase of the two terms, and normalization demands that $a^2+b^2=1$. The source thus produces pairs of horizontally polarized photons with probability $a^2$ and pairs of vertically polarized photons with probability $b^2$. The relative magnitudes of $a$ and $b$ are adjusted by rotating the half-wave plate that controls the pump polarization. The relative phase is controlled by tilting the quartz plate, and we wish to adjust this phase to be $\phi=0$.[8] As described in Appendix A, when performing Hardy's test, a near optimal violation of local realism can be accomplished using either of the states

$$|\psi_1\rangle = \sqrt{0.8}|H\rangle_A|H\rangle_B + \sqrt{0.2}|V\rangle_A|V\rangle_B, \tag{6a}$$

$$|\psi_2\rangle = \sqrt{0.2}|H\rangle_A|H\rangle_B + \sqrt{0.8}|V\rangle_A|V\rangle_B. \tag{6b}$$

For $|\psi_1\rangle$ the analysis angles used are $\alpha\approx 55°$ and $\beta\approx 71°$, and for $|\psi_2\rangle$ the analysis angles are $\alpha\approx 35°$ and $\beta\approx 19°$.

At each detection station the photons first pass through a half-wave plate (we use zero-order broadband wave plates) and then through a broadband polarizing beam splitter. Rotating the wave plate in front of the beam splitter is equivalent to rotating the polarization axis of the beam splitter. Light emerging from the beam splitter is collected by a lens and focused into a multimode fiber that directs the light to a single photon counting module (SPCM). The SPCMs have RG780 filters in front of them, which pass the downconverted light but block scattered pump photons.

We used two different detection schemes in our experiments: a two-detector scheme and a four-detector scheme. The four-detector scheme is shown in Fig. 2; in the two-detector scheme we use only the detectors monitoring the transmission ports of the beam splitters. The unit that we use to count the photons contains four independent SPCMs packaged together. Each SPCM outputs a 5-V electrical pulse for every detected photon. In the four-detector scheme we measure coincidences between four combinations of detectors. For example, $N_{AB}$ is the number of coincidence counts between detectors $A$ and $B$ in a given time interval. We also measure the coincidence counts $N_{AB'}$, $N_{A'B}$, and $N_{A'B'}$. In the two-detector scheme we measure only $N_{AB}$. Coincidence counting is done using four time-to-amplitude converter/single-channel-analyzer (TAC/SCA) units. We use a 3.5-ns coincidence window. A PCI-card, eight-channel counter in a computer does the actual counting. Four channels measure the coincidence counts, while the other four channels measure the singles counts on each detector: $N_A$, $N_{A'}$, $N_B$, and $N_{B'}$.

Further information about our experimental apparatus, especially details about coincidence counting, can be found in Refs. 23 and 24. For further details about the two-crystal source, the reader is referred to Refs. 8, 15, and 22.

### B. Measuring probabilities

Given the measured count rates, we need to calculate the probabilities that determine $H$ using Eq. (4). In the four-detector scheme the probabilities are given by the ratio of the measured coincidence counts on the transmitted beams to the total number of coincidences,

$$P(\theta_{Ai},\theta_{Bj}) = \frac{N_{AB}(\theta_{Ai},\theta_{Bj})}{N_{AB}(\theta_{Ai},\theta_{Bj}) + N_{AB'}(\theta_{Ai},\theta_{Bj}) + N_{A'B}(\theta_{Ai},\theta_{Bj}) + N_{A'B'}(\theta_{Ai},\theta_{Bj})}. \tag{7}$$

For a given choice of angles we can simultaneously measure all of the needed coincidence counts to determine $P(\theta_{Ai},\theta_{Bj})$ when using the four-detector scheme. Because we need to measure four probabilities to determine $H$, we need to make measurements at four different combinations of wave-plate angles.

In the two-detector scheme we only measure one coincidence count rate at a time. We need four coincidence rates to determine the probability (corresponding to the angles of interest as well as the perpendicular combinations of angles), so we must use four wave-plate settings for each probability. The measured probability is given by

$$P(\theta_{Ai},\theta_{Bj}) = \frac{N_{AB}(\theta_{Ai},\theta_{Bj})}{N_{AB}(\theta_{Ai},\theta_{Bj}) + N_{AB}(\theta_{Ai},\theta_{Bj}^{\perp}) + N_{AB}(\theta_{Ai}^{\perp},\theta_{Bj}) + N_{AB}(\theta_{Ai}^{\perp},\theta_{Bj}^{\perp})}. \tag{8}$$

Determining $H$ with the two-detector scheme requires measurements to be preformed at 16 different combinations of wave-plate angles. Thus obtaining results from the two-detector scheme that are comparable to those from the four-detector scheme requires a data acquisition time that is four times longer.

### C. Tuning the state

The key to the experiment is aligning the source to produce a state that closely approximates one of the states given in Eq. (6). We start by adjusting the pump polarization to be horizontal, so that it pumps only one of the two crystals, and the downconversion is vertically polarized. We align detectors $A$ and $B$ for maximum coincidence counts, as described in Ref. 23.

We then rotate the pump polarization to vertical to produce horizontally polarized downconversion from the other crystal. We use the fact that when the downconversion crystals are tilted, they are very sensitive to tilt in one direction, but fairly insensitive to tilt in the opposite direction. With the pump vertically polarized we adjust only the vertical tilt of the crystal pair to maximize the production of horizontally polarized photon pairs; this adjustment does not significantly affect the production of vertically polarized pairs from the other crystal. We now insert iris diaphragms to define the paths of the signal and idler beams, insert the wave plates and polarizers in front of the detectors, and align detectors $A'$ and $B'$. These detectors are coarsely aligned by shining laser light backward through the coupling fibers, as described in Ref. 23.

To tune the state we set the detector wave plates so that $N_{AB}$ registers $HH$ coincidences, and $N_{A'B'}$ registers $VV$ coincidences. We adjust the wave plate in the pump beam so that the ratio of these coincidences is 4:1 to produce state $|\psi_1\rangle$ (1:4 to produce state $|\psi_2\rangle$). If we use only two detectors, we must adjust the detector wave plates to sequentially record the $HH$ and $VV$ coincidences to adjust their ratio. We then set the detector wave plates to measure $N_{AB}(-\alpha,\alpha)$ and adjust the tilt of the quartz plate in the pump beam [hence adjusting $\phi$ in Eq. (5)] to minimize these coincidences. The advantage of using four detectors when doing this procedure is that we can monitor $P(-\alpha,\alpha)$ in real time while performing this adjustment. Ideally we wish this probability to be less than 1%, but at this point in the alignment we are willing to settle for a few percent. Now the state should be fairly well tuned, and we should be able to verify that $P(\beta,\alpha^{\perp})$ and $P(-\alpha^{\perp},-\beta)$ are both on the order of a few percent. Fine-tuning is done by iteratively adjusting the wave plate and the quartz plate in the pump beam, as well as varying the measurement angles over a few degrees, all with the goal of keeping the measured values of $P(-\alpha,\alpha)$, $P(\beta,\alpha^{\perp})$, and $P(-\alpha^{\perp},-\beta)$ as low as possible while increasing $H$. As described in Appendix A, it is possible to adjust the measurement angles to increase $H$ with little measurable affect on $P(-\alpha,\alpha)$, $P(\beta,\alpha^{\perp})$, and $P(-\alpha^{\perp},-\beta)$. Note that the angles are not independent; if we change $\alpha$ to help minimize $P(-\alpha,\alpha)$, for example, we must make consistent changes when measuring $P(\beta,\alpha^{\perp})$ and $P(-\alpha^{\perp},-\beta)$.

## IV. RESULTS

When taking data we set the detector wave plates to the desired angles and then collect coincidence data for a given integration time. We collect 10 measurements of coincidence data before changing the wave-plate settings and repeating the process. These 10 measurements at each combination of angles allow us to compute 10 values for $H$ and hence determine the mean and standard deviation of $H$. The total coincidence rate between the signal and idler beams is typically 400 counts per second, and we use integration times ranging from 10 to 60 s. The total time for an experiment using the four-detector scheme and a 10-s integration time is about 7 min.

Our best results are obtained with the four-detector scheme. We find $H=0.1178\pm0.0016$, where the quoted error is the standard deviation. This result violates the inequality $H\leqslant 0$ set by local realism by 73 standard deviations. This experiment used a 45-s integration time and the individual probabilities were $P(\beta,-\beta)=0.147$, $P(-\alpha,\alpha)=0.012$, $P(\beta,\alpha^{\perp})=0.008$, and $P(-\alpha^{\perp},-\beta)=0.009$. In a more typical run with a 20-s integration time we find $P(\beta,-\beta)=0.153$, $P(-\alpha,\alpha)=0.021$, $P(\beta,\alpha^{\perp})=0.012$, $P(-\alpha^{\perp},-\beta)=0.013$, and $H=0.1081\pm0.0033$, for a 32 standard deviation violation of local realism. These two values for $H$ are not the same because they were obtained on different days, with slightly different tunings of the quantum state.

These results were obtained using state $|\psi_2\rangle$, but we have obtained essentially the same results using state $|\psi_1\rangle$. Our best results with the two-detector scheme violated local realism by 18 standard deviations. This reduction in the amount of violation was primarily because the two-detector scheme takes four times as long to acquire the data, so we tended to use shorter integration times.

Our technique of obtaining 10 measurements of $P(-\alpha,\alpha)$, then 10 measurements of $P(-\beta,\beta)$, etc., makes it possible for a slow drift in the apparatus to affect the statistics of $H$. In practice, we find that a slow drift is not a significant issue in our experiment. We tested for drift by performing our experiment 11 times over a 3.5-h interval, without changing the alignment or measurement angles. All of the measured values of $H$ were the same to within the error estimate. This observation is not surprising; the experiment does not require interferometric precision, so we do not expect it to be particularly sensitive to drift. The alternative data collection technique would be to rotate the wave plates after every measurement, which would greatly increase the data acquisition time.

The results we have given were obtained by two undergraduate students as part of a summer project. Since then we have implemented this experiment in an undergraduate teaching laboratory. Four groups of students each spent two three-hour lab periods working on the experiment. The detector alignment was done before the students came to the lab. After familiarizing themselves with the apparatus, the students' primary task was to explore how tuning the state (using the pump wave plate and the quartz plate) and adjusting $\alpha$ and $\beta$ affected the measurements. Each group explored the behavior of both states $|\psi_1\rangle$ and $|\psi_2\rangle$, and all groups were able to obtain at least a 10 standard deviation violation of local realism.

## V. CONCLUSIONS

There are now several experiments on fundamental aspects of quantum mechanics using individual photons that have been performed in undergraduate laboratories. In addition to Hardy's test of local realism, there are tests of a Bell inequality,[8] showing the existence of photons,[23] single photon interference,[24,25] and the quantum eraser.[26,27] These experiments have been reproduced in several colleges and universities. The technology to perform them continues to improve and implementing them in an undergraduate laboratory continues to become easier.

We have experience with all of these experiments in our laboratory and can offer some comments on their relative difficulty. We have found that the experiment that demonstrates the existence of photons is the most straightforward. This relative ease is because it uses a simple source (a single type-I downconversion crystal), has very high count rates, and is straightforward to align. The next most difficult experiments are those that add one more level of complexity: either a polarization entangled source or an interferometer. Hardy's test of local realism and the test of a Bell inequality both require a polarization entangled source, and single photon interference requires aligning an interferometer. The quantum eraser is the most difficult because it involves using both a polarization entangled source and an interferometer.

## ACKNOWLEDGMENTS

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## APPENDIX A: QUANTUM STATE

Consider the quantum state

$$|\psi\rangle = a|H\rangle_A|H\rangle_B + b|V\rangle_A|V\rangle_B, \tag{A1}$$

where $H$ refers to a horizontally polarized photon, $V$ refers to a vertically polarized photon, $a$ and $b$ are real numbers, and normalization demands that $a^2+b^2=1$. If the light emerging from the source is in this state, then the probability that Alice will measure her photon to be polarized along $\theta_{Ai}$ and Bob will measure his photon to be polarized along $\theta_{Bj}$ is given by

$$P(\theta_{Ai},\theta_{Bj}) = |({}_A\langle\theta_{Ai}|{}_B\langle\theta_{Bj}|)|\psi\rangle|^2. \tag{A2}$$

Equation (A2) can be evaluated by noting that the state of a photon polarized at an angle $\theta$ with respect to the horizontal can be written as

$$|\theta\rangle = \cos\theta|H\rangle + \sin\theta|V\rangle, \tag{A3}$$

so

$$\begin{aligned}({}_A\langle\theta_{Ai}|{}_B\langle\theta_{Bj}|)|\psi\rangle &= ({}_A\langle H|\cos\theta_{Ai} + {}_A\langle V|\sin\theta_{Ai})\\ &\quad\times({}_B\langle H|\cos\theta_{Bj} + {}_B\langle V|\sin\theta_{Bj})\\ &\quad\times(a|H\rangle_A|H\rangle_B + b|V\rangle_A|V\rangle_B).\end{aligned} \tag{A4}$$

After simplifying Eq. (A4) we find

$$P(\theta_{Ai},\theta_{Bj}) = (a\cos\theta_{Ai}\cos\theta_{Bj} + b\sin\theta_{Ai}\sin\theta_{Bj})^2. \tag{A5}$$

We are interested in finding a state and a set of angles that satisfy observations 1, 2′, 3′, and 4 and for which $H>0$, with $H$ defined in Eq. (4). It is easily verified from Eq. (A5) that if $a=\sqrt{0.8}$, $b=\sqrt{0.2}$, $\alpha=55°$, and $\beta=71°$, the probabilities in $H$ are $P(\beta,-\beta)\cong 0.09$ and $P(-\alpha,\alpha)\cong P(\beta,\alpha^\perp) = P(-\alpha^\perp,-\beta)\cong 0$,[28] yielding $H=0.09>0$, which violates local realism. The same probabilities are obtained with $a=\sqrt{0.2}$, $b=\sqrt{0.8}$, $\alpha=35°$, and $\beta=19°$.

The parameters we have given yield nearly the maximum possible violation of the inequality $H\leqslant 0$, assuming that the angles are constrained such that $P(-\alpha,\alpha)=P(\beta,\alpha^\perp) = P(-\alpha^\perp,-\beta)=0$.[11] It is possible to obtain larger values of $H$ if we are willing to relax the constraint that these probabilities are all equal to zero. Assume, for example, that we are willing to allow these probabilities to be greater than zero but less than 1%. With $a=\sqrt{0.8}$, $b=\sqrt{0.2}$, $\alpha=59°$, and $\beta=80°$, $P(-\alpha,\alpha)$, $P(\beta,\alpha^\perp)$, and $P(-\alpha^\perp,-\beta)$ are all less than 1%, and $P(\beta,-\beta)=0.165$, yielding $H=0.140$. Experimentally we found it extremely difficult to obtain probabilities less than 1%, and were unable to get all three of these probabilities simultaneously less than 1% (although others have done so[13–15]).

## APPENDIX B: EQUIPMENT

Much of the equipment we use is the same as that described in Refs. 8,23. However, some of the technology has advanced, so we provide updated information here and on our Web site.[24]

We use essentially the same crystals as described in Ref. 8 (two crystals glued together by the manufacturer), except that ours are 0.5 mm thick rather than 0.1 mm thick.[29] We have also performed this experiment with two individually mounted crystals placed back to back, and the results obtained with these crystals were almost as good as those we have described. Because the separate crystals involve more work to align, we recommend using crystals that are glued together.

The laser we use is a 50-mW, 405-nm laser diode with a circularized output beam (model IQ1C50) from Power Technologies.[30] Although this laser is only one year old, newer technology has already replaced it. Currently Power Technologies has 185-mW lasers available.

We use a Perkin-Elmer, four-channel, single photon counting module.[31] This module needs external 2, 5, and 30 V power supplies. Because the price of this four-channel unit is only slightly more than the price of two individual counters, we recommend purchasing this unit, which allows one to implement (or later upgrade to) the four-detector scheme.[32]

We already had motorized rotation stages (model PR50PP) and a computer interfaced controller (model ESP300) for the detector half-wave plates from Newport Corporation.[33] These stages and controller were a luxury—they make life easier for the experimenter, but are not necessary.

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[30] http://www.powertechnology.com/; total price $7,165 with power supply and mounting bracket.
[31] Model SPCM-AQ4C, available from Pacer Components, http://www.pacer.co.uk/. We recommend purchasing the input–output adapter card (SPCM-AQ4C-IO), which greatly simplifies getting power into the unit and getting the signals out. The price with the card is $9,250.
[32] With the four-channel unit, the primary additional expense to implement the four-detector scheme is the need to have four TAC/SCA units to perform coincidence counting, rather than just one.
[33] http://www.newport.com/. Although we have not used it, we note that Newport has just released the NSR1 rotation stage, which is significantly cheaper. The total cost for two stages and the necessary controller electronics is approximately $2,600. The NSR1 only has a resolution of 1°, but that should be sufficient for this experiment.